Volume 4, Number 4 (October, 2011) p. 610-641 • ISSN 1983-4195

REVISTA IBRACON DE ESTRUTURAS E MATERIAIS
IBRACON STRUCTURES AND MATERIALS JOURNAL

Reinforced concrete waffle flat slabs under shearing

Lajes lisas nervuradas de concreto armado ao cisalhamento


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Abstract

The structural systems with waffle flat slabs are the most used in Brazil when the main requirement is flexible "layouts" and long spans with reasonable economy. However, few studies have been developed in Brazil regarding the behavior of these slabs when the flexural resistance is satisfactory and the shear in the ribs and punching resistances become competitive. This work shows the experimental analysis of 8 two-way reinforced concrete waffle flat slabs under centered load. The dimensions of the slabs were the same and equal to 1800 mm x 1800 mm x 140 mm. The ribs were 80 mm (height) by 50 mm (width) and the compressive concrete strength was approximately 40 MPa. The experimental results were compared to those estimated by the Brazilian code NBR 6118:2003. It was verified that the resistance of the ribs is not satisfactorily estimated by the code, which excessively underestimates the results for ribs with and without shear reinforcement.

Keywords: reinforced concrete, waffle slab, flat slab, shearing, punching.

Resumo

Os sistemas estruturais com lajes lisas nervuradas são os mais utilizados no Brasil quando a exigência principal é a disponibilidade de "layouts" flexíveis e grandes vãos com razoável economia. Entretanto, poucos estudos foram realizados no Brasil considerando o comportamento destas lajes quando a resistência à flexão é satisfatória e as resistências ao cisalhamento nas nervuras e ao puncionamento tornam-se concorrentes. Este trabalho traz as análises experimentais de 8 lajes lisas nervuradas bidirecionais de concreto armado sujeitas a carregamento centrado. As dimensões das lajes foram constantes e iguais a 1.800 mm x 1.800 mm x 140 mm. As nervuras apresentaram 80 mm de altura por 50 mm de base e a resistência do concreto à compressão foi de aproximadamente 40 MPa. Os resultados foram comparados aos estimados pela norma brasileira NBR 6118:2003 [1]. Verificou-se que a resistência das nervuras não é satisfatoriamente estimada pela norma, subestimando demasiadamente os resultados para as nervuras sem e com armadura de cisalhamento.

Palavras-chave: concreto armado, laje nervurada, laje lisa, cisalhamento, punção.

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Received: 28 Aug 2010 • Accepted: 17 May 2011 • Available Online: 07 Oct 2011

1. Introduction

The increasing use of larger spans and the masonry walls directly on the slabs positing, mainly because of the architecture requirements, are increasingly common on the floors of buildings. The massive use of flat slabs in such cases leads to higher thicknesses, the structure may become uneconomic, as part of the bearing capacity of the slab is used to resist to its self weight. In this case, the use of ribbed flat slabs is an attractive alternative because it is a structural system that consists of slabs supported directly on the columns (without beam) through a massive region, and uni or bidirectional ribs, where part of the concrete below the neutral line is eliminated, being replaced by a filling material, which are commonly used blocks of expanded polystyrene (EPS), or removable formwork, reducing the weight of the slab and permitting larger spans.
The structural system in flat ribbed slabs has several advantages over the conventional solid slabs (beams and columns) in the same span, can cite the reduction in the formwork amount, the lower consumption of materials and manpower, and lower self weight, creating a relief in the foundations, with or without inert material between the ribs. There is also greater freedom and flexibility to adapt the internal space (in the absence of beams) and is indicated primarily for residential buildings, hospitals and garages, ease the passage of common and special pipes lines.
Despite the advantages mentioned above, the elimination of the beams carries some disadvantages, such as increasing vertical displacements compared to conventional slabs with the same span, decreased overall stability of the structure due to horizontal actions, the possibility of failure by punching, consequently, progressive collapse, and the possibility of shear failure of the ribs close to the solid concrete area. The failure by punching can occur due to concentrated loads or distributed in small areas, directly on the slabs. According to Souza and Cunha (1998) [2], this type of failure is so fragile and sudden (without warning), it usually occurs before the flexure reinforcement reaches the bending yield stress, which may cause progressive collapse of the structure. Thus, the objective of this study was to evaluate the behavior of 8 two-way flat slabs of reinforced concrete with shear reinforcement in the ribs and punching reinforcement in the massive region, subjected to centered loads.

2. Experimental programm

2.1 Slabs characteristics

Tests were conducted in 8 two-way square ribbed flat slabs of reinforced concrete with 1800 mm side and 140 mm thick. Loads were applied from the bottom of the slabs surface through a square metal plate with 85 mm side and 50 mm thick, simulating the action of a column. Reinforcement bending were the same for all slabs, consisting of bars of 6.0 mm and 12.5 mm diameter bars in the *x* direction and 12.5 mm in diameter in the *y* direction, providing a geometric rate of reinforcement bending (r) of approximately 1.40%, determined according to the recommendations of the CEB-FIP MC90 [3]. The main variables considered were the types of shear reinforcement in the ribs, consisting of trusses, vertical closed stirrups and open stirrups inclined at 45 degrees and the use of stirrups inclined at 45 degrees with punching reinforcement in the solid region. Table 1 shows the main characteristics and dimensions of the slabs are shown in Figure 1.
The main reinforcement, located on the upper surface of the slab, was composed of 21 bars of 12.5 mm and 6 bars of 6 mm in diameter in the *x* direction and bars 21 bars of 12.5 mm in diameter in the *y* direction. In the bottom surface of the slabs were placed only distribution reinforcement positioned longitudinally and transversely, composed of 12 bars of 4.2 mm in diameter in each direction, and arranged two bars per rib. Figure 2 shows the positioning of the flexural reinforcement. For slabs with shear reinforcement in the ribs were used three different types of elements, consisting of trusses (TR 8644), in order to investigate the efficiency of the diagonals in the shear resistance, vertical closed stirrups and open stirrups inclined of 45 degrees, and all the stirrups were made of 4.2 mm in diameter. Figure 3 shows the three types of reinforcement used.
Regarding the punching reinforcement in the solid area were used open stirrups inclined of 45 degrees, from bars with a diameter of 6.3 mm and arranged in three layers distributed in cross. The

Table 1 – Slabs main characteristics

Slab	ρ (%)	d (mm)	f_c (MPa)	Shear reinforcement: Rib (x direction)	Shear reinforcement: Rib (y direction)	Punching
L1	1.27	120	41	-	-	-
L2	1.44	106	37	truss	Vertical stirrup	-
L3	1.37	111	38	truss	Vertical stirrup	-
L4	1.29	118	39	truss	Vertical stirrup	-
L5	1.33	115	38	Vertical stirrup	Vertical stirrup	-
L6	1.47	104	40	truss	Vertical stirrup	Inclined stirrup
L7	1.36	112	41	Vertical stirrup	Vertical stirrup	Inclined stirrup
L8	1.41	108	39	Inclined stirrup	Inclined stirrup	Inclined stirrup

ρ: flexural reinforcement geometrical rate; f_c: compressive strength of concrete.

choice for this type of shear reinforcement was due to the ease of installing them on the slab, but also outperforms the vertical stirrups, considering the ultimate resistance of the elements, according to Oliveira (1998) [4]. Figure 4 shows the placement of shear reinforcement in the ribs and in the solid area.

2.2 Slabs monitoring

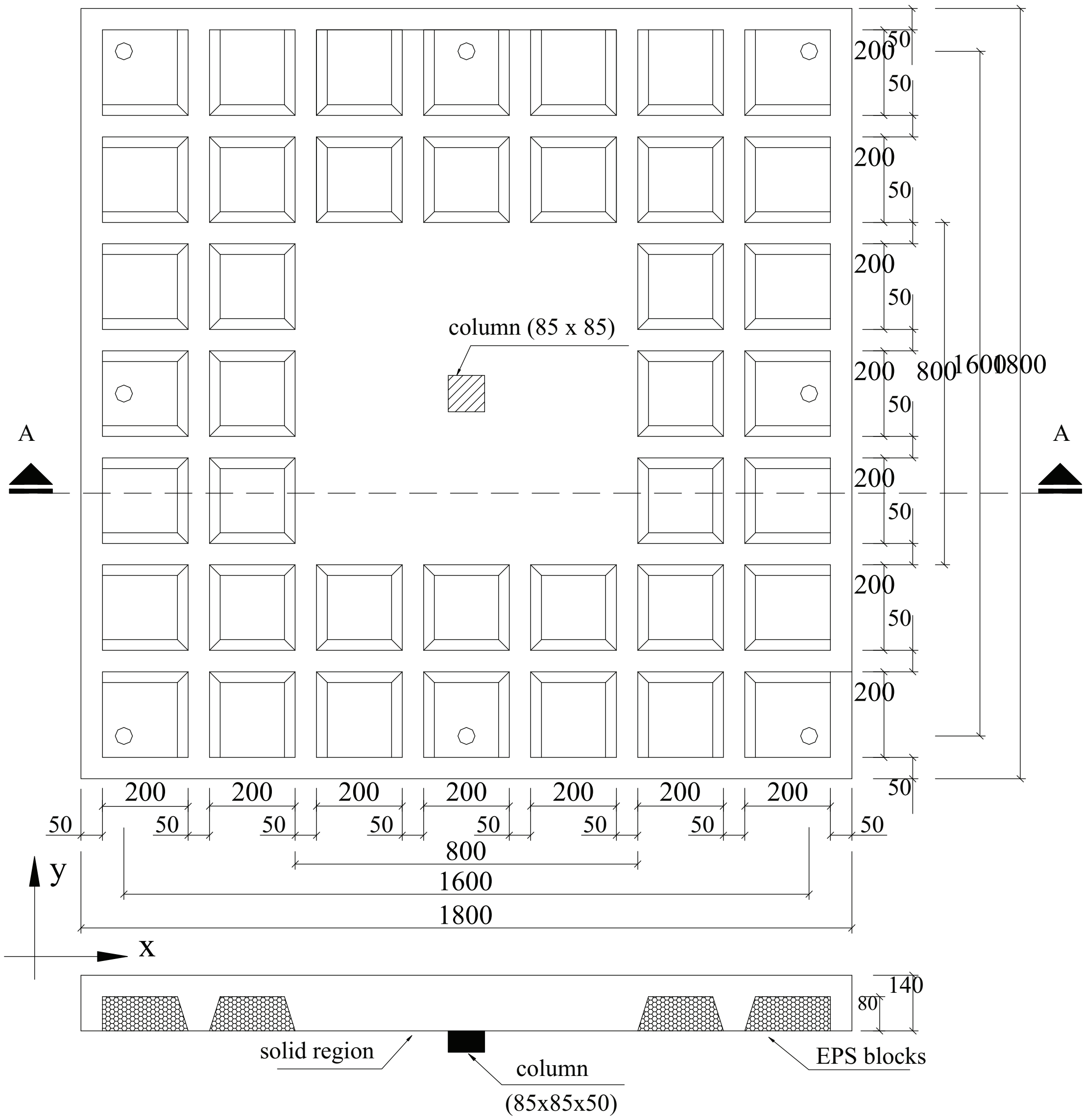
2.2.1 Displacements

The vertical displacements were measured using 7 dial gauges positioned in the middle of the span, distributed in two directions (*x* and *y*), spaced 174.5 mm apart and in contact with the upper surface of the slabs. Figure 5 shows the scheme of the gauges positioning on the slabs, indicated by the letter D.

2.2.2 Concrete Surface

To measure the concrete strains were used strain gauges (EERs) fixed to the bottom surface of all slabs, and subsequently connected to the equipment data acquisition (*Spider 8*). Four EERs were used (C1, C2, C3 and C4) on the slab without shear reinforcement (L1) and on the slabs where the shear reinforcement was different in the *x* and *y* directions (L2, L3, L4 and L6), whereas in slabs with the same shear reinforcement in both directions (L5, L7, L8) were fixed only two EERs (C1 and C3).

Figure 1 – Plan view and cross section of the tested slabs (section AA)



The EERs were positioned in the region 55 mm away from the face of the column, and only in the tangential direction, this position is justified by the fact of having a predominance of tangential stresses on the radial stresses in this type of structural system (Oliveira, 1998). The positioning of the strain gauges in the concrete surface is shown in Figure 6.

2.2.3 Flexural reinforcement

The flexural reinforcement strains were monitored in the same way that those on the concrete surface, taking into account the type of shear reinforcement ribs in the *x* and *y* directions, but always in the direction of the tangential stresses of the slabs as well as the tangential strains are much more important than the radial. Each bar was instrumented using one strain gauge at half height of the bar. The slabs L1, L2, L3, L4 and L6 showed the same position and amount of strain (E1, E2, E3 and E4) and similarly the slabs L5, L7 and L8 showed the same position and amount of strain gauges (E1 and E3). Figure 7 shows the positioning of the strain gauges in the flexural reinforcement and a detail of the gauges installation on the bars surfaces.

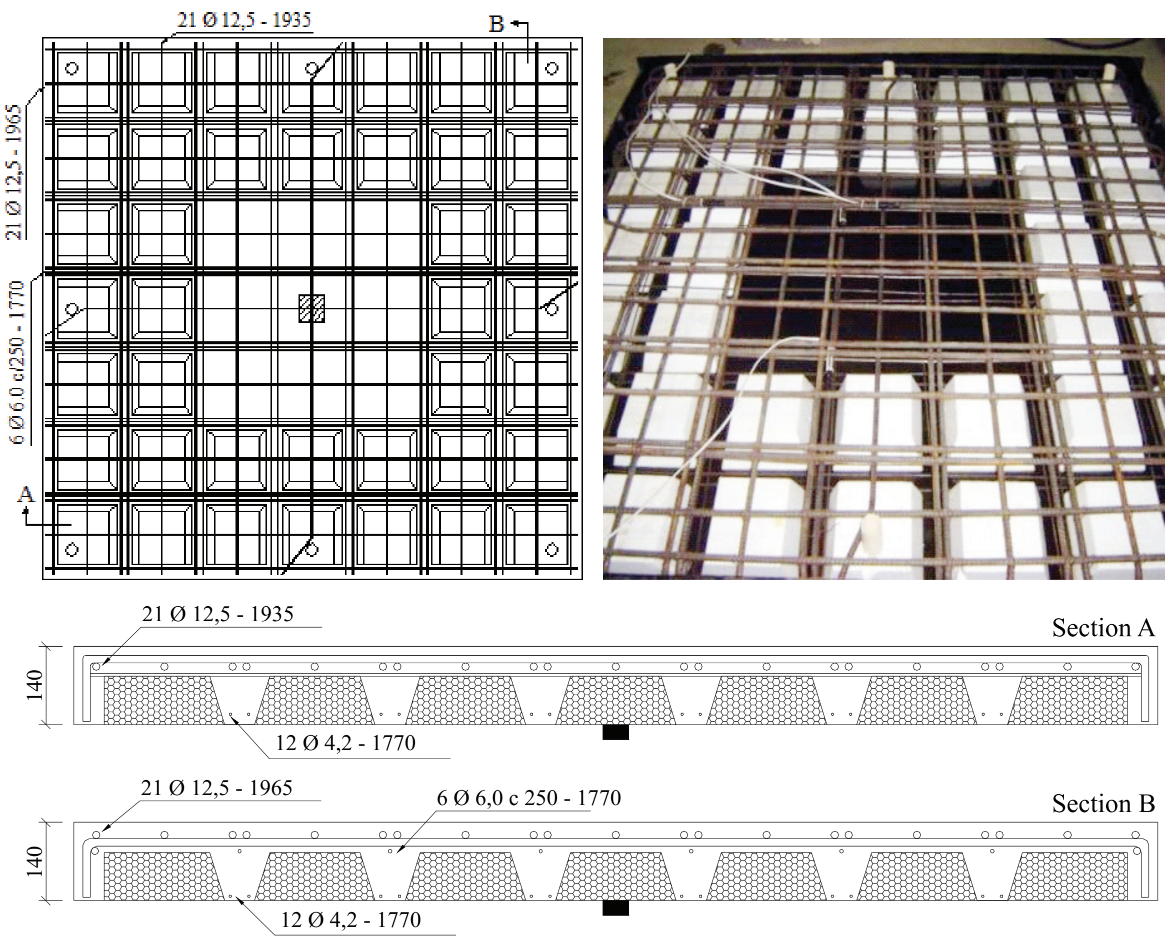
2.2.4 Shear Reinforcement

To measure the shear reinforcement strains in the ribs and solid area were set EERs of the same type used in the flexural reinforcement. In shear reinforcements composed of trusses was placed a strain gauge on the tensile diagonal and in the shear reinforcement composed of stirrups, both vertical closed and open inclined at 45 degrees was set a strain gauge on a leg of the stirrup, positioned at half height (Figure 8).

2.3 Loading system

The test system consisted of slabs supported on all four sides by steel beams reaction, simulating a continuous support of the slabs. The load was applied through a hydraulic cylinder on a steell plate simulating the action of a column, powered by a hydraulic pump whose intensity was measured by a load cell coupled to a digital display. The loading applied to the tested slabs was transmitted to the laboratory slab by reaction of 8 steel ties.
After the assembly process of the system, the loading was applied upward from the slabs bottom surface, adopting a load increase of

Figure 2 – Details of the flexural reinforcements



approximately 10% of the estimated failure load. For each load increment the vertical displacements were measured through seven dial gauges and readings of the strains in reinforcement and concrete were performed with two modules of the equipment data acquisition *Spider 8*. The details of the test system are shown in Figures 9 and 10.

3. Results

3.1 Materials

The mechanical properties of concrete were determined from tests of compressive strength, tensile strength by diametrical compression and modulus of elasticity, according to NBR 5739:1994 [5], NBR 7227:1994 [6] and NBR 8522:1984 [7], respectively. The results presented in Table 2 represent the average of the three proof cylinders tested for each slab in their ages. The characteristics of the steels used in this study were obtained from axial tensile test, according to NBR 6152:1992 [8] and are presented in Table 3.

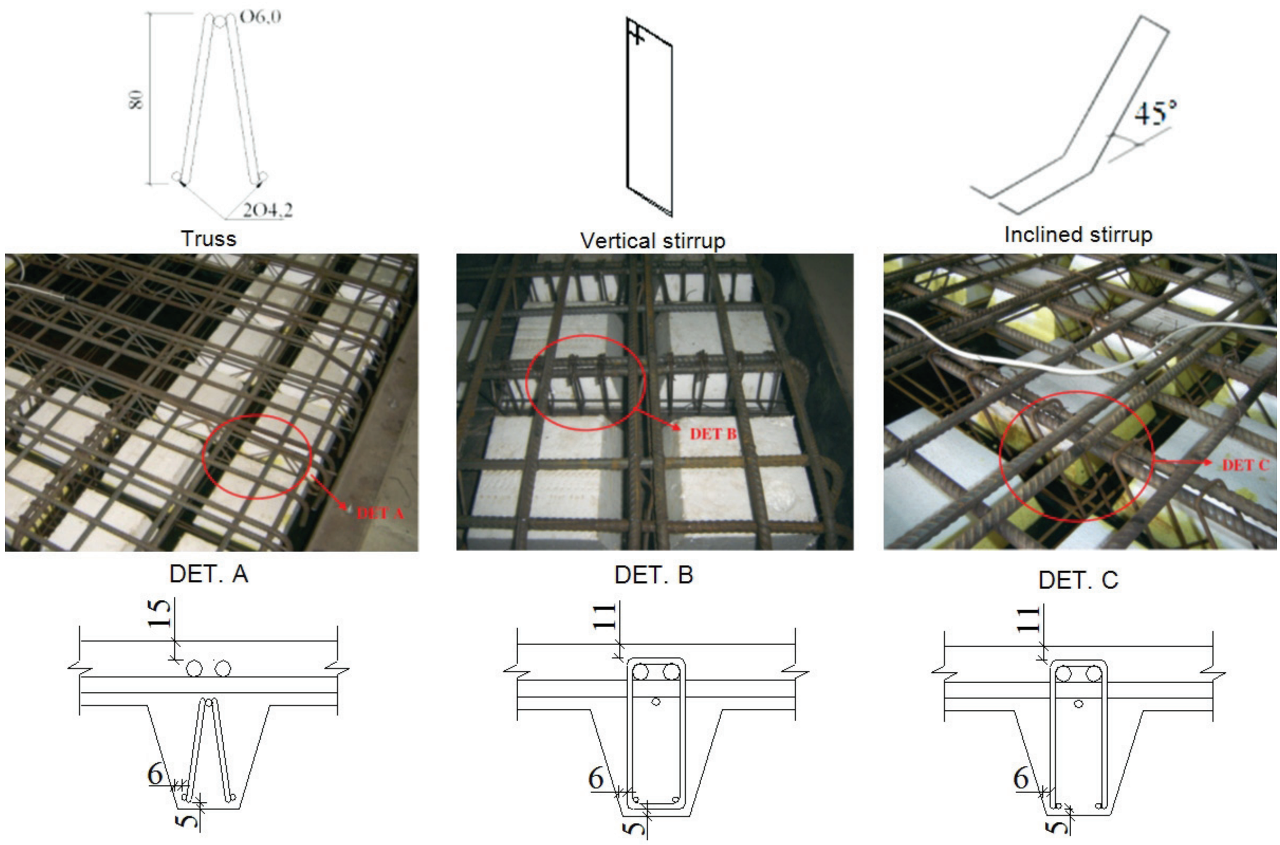
3.2 Displacements

The dial gauges were distributed in two directions (*x* and *y*) in order to compare the behavior of the slabs due to the variation of shear reinforcement in the ribs and the use of punching reinforcement. Figure 11 shows the vertical displacements observed in the slabs. Figure 12 shows the vertical central displacements (D4) of all slabs for each load applied. The vertical displacements of the slabs were different for the corresponding points in both directions, indicating possible problems of symmetry of the column or support, and even the occurrence of differential accommodation of the test system during the tests. The greatest discrepancies were found in L4 slab for the meters D1, D2, D6 and D7. But the central gauge (D4), placed on loaded region, had the highest vertical displacements for all slabs, as expected. The maximum displacement (Figure 12) shows a similar behavior between the slabs, with three slabs with punching reinforcement (L6, L7 and L8) showed the largest final displacements.

3.3 Concrete strains

Only the tangential strains were measured, once they are higher than the radial ones (Oliveira, 1998 and Smith, 2004 [9]). The slabs L6 and L7 were the ones with the largest strains (4.19 ‰ and 3.64 ‰, respectively), indicating the occurrence of t concrete

Figure 3 – Shear reinforcement in the ribs

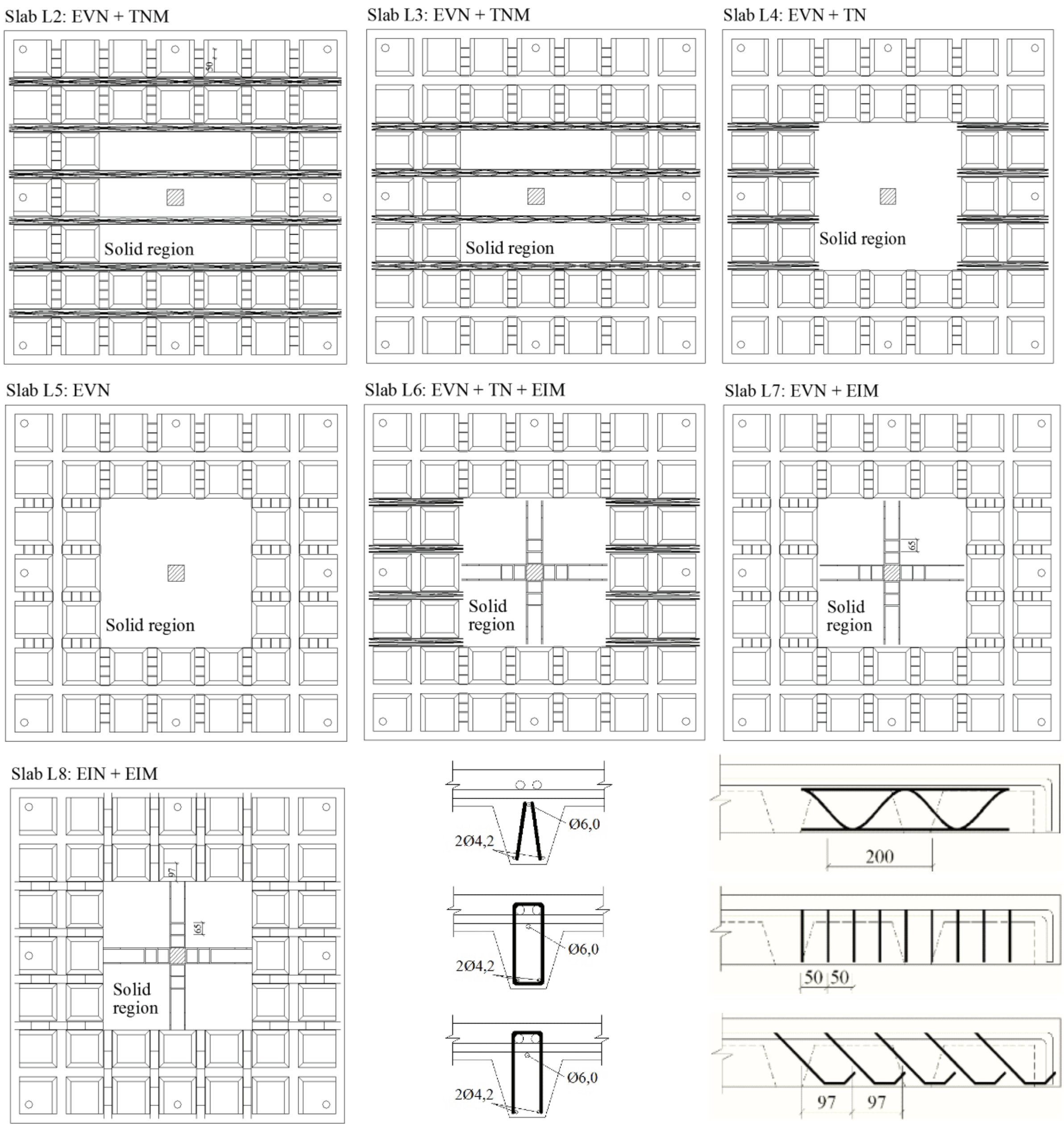


crushing in these slabs. Figure 13 shows the maximum strains of the concrete for all slabs.

3.4 Flexural reinforcment strains

The largest strains were recorded near the column (E1) in the slab L7, with the flexural reinforcement showing maximum strain of 4.38 ‰, reaching the yielding (strains greater than 2.5 ‰). In addition L7 slab, the flexural reinforcements of the slabs L1 (3.76 ‰), L2 (2.72 ‰) and L6 (3.79 ‰) also yielded. Figure 14 shows the maximum strain in the flexural reinforcement of the slabs.

Figure 4 – Shear reinforcement (ribs and solid region)



EVN: vertical closed stirrups in the rib: Ø4.2 e 50; EIN: inclined open stirrups in the rib: Ø4.2 e 97; EIM: inclined open stirrups in the solid region: Ø6.3 e 65; TN: truss in the rib: Ø4.2 e 200; TNM: truss in the rib and solid region: Ø4.2 e 200.

Figure 5 – Dial gauges arrangement on the slabs

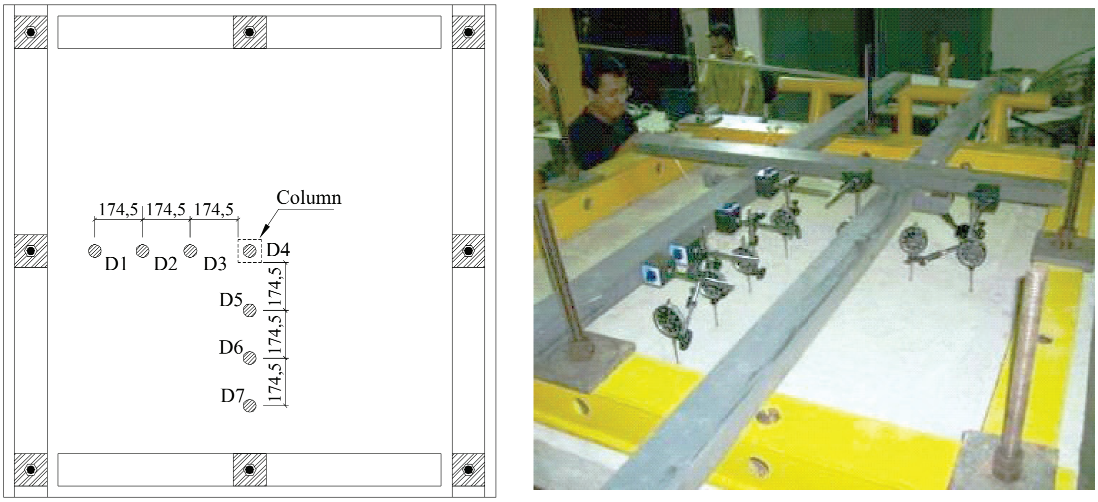
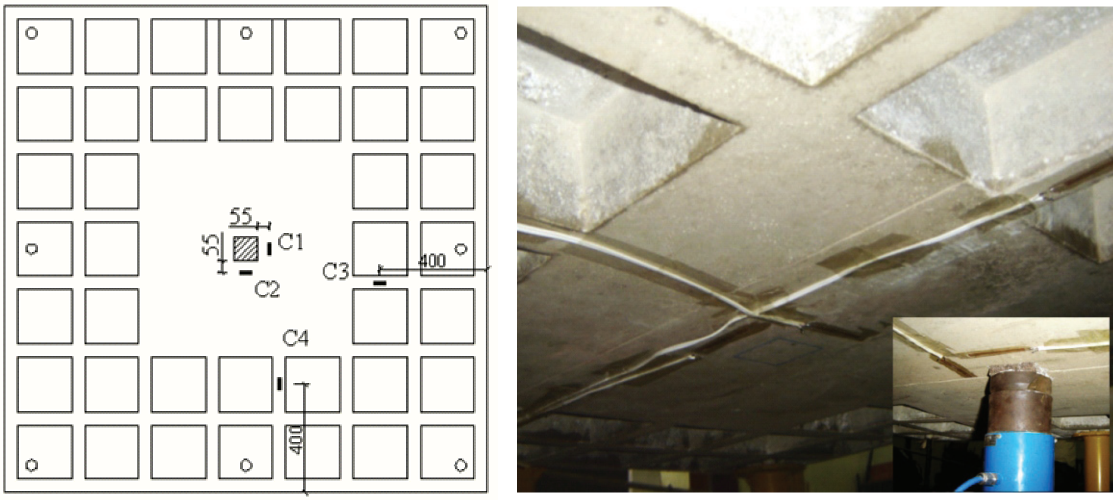


Figura 6 – Strain gauges arrangement on concrete surface



3.5 Shear reinforcement strains

The strains in ribs shear reinforcement were much smaller than the yield strain of steel (ε_{ysw}=4.7). However, in some slabs (L6 and L7), there was a change in the shear failure mode of the ribs for flexure, being the effective the inclined reinforcement (truss and open stirrups inclined at 45 degrees) for slabs providing greater ductility. Figure 15 shows the strains of the shear reinforcement in two of the slabs tested. In relation to punching reinforcement (45 degrees inclined stirrup), despite having been requested enough, had no stirrup yielding (strain greater than 4.43 ‰), and reached the maximum deformation of 3.34 ‰, observed in the slab L8 (the slab with inclined stirrups in the ribs), i.e. performed better when coupled with the use of inclined stirrups in the ribs. It can be seen (Figure 16) that with approximately 80% of tensile strength, the reinforcements began to strain in a non linear manner in relation to the applied loads, indicating that the yielding was not so far and confirming the gain of ductility of the slabs obtained with the introduction of shear reinforcement.

3.6 Loads and failure modes

The use of shear reinforcement in the ribs did not cause gains in ultimate load, because sometimes the flexural or punching strength had been reached, making it impossible for the shear reinforcement were requested in order to provide an increase in the failure loads. In relation to punching reinforcements, they provide significant gains for failure load, compared to the reference slab (L1), giving to the slabs, with this kind of reinforcement, a more ductile behavior. Table 4 shows the loads and modes of failure observed in the slabs. In situations where it was not possible to clearly observe the failure mode, the criterion used for classification was based on that presented by Oliveira (1998) [4], where the flexure mode occurs for $P_u/P_{flexure} > 1.0$, with strains on the flexural reinforcement bars greater or equal to yield strain characteristic of the steel, and the shear modes (punching and shear in the ribs) for values less than 1.0. $P_{flexure}$ here is the estimated failure load by flexure and P_u is the experimental failure load. Thus, the failure modes of the

Figure 7 – Strain gauges arrangement in the flexural reinforcement

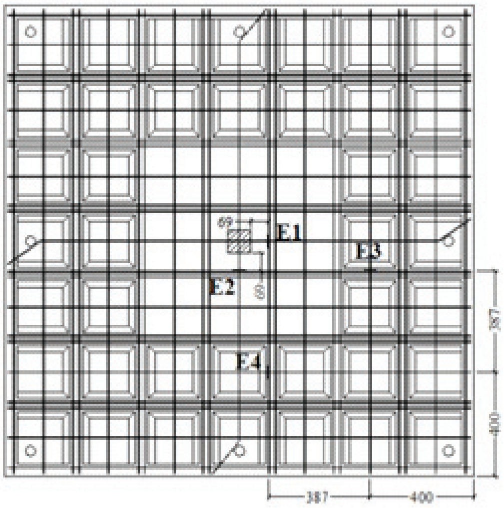


Figure 8 – Strain gauges arrangement in the shear reinforcement

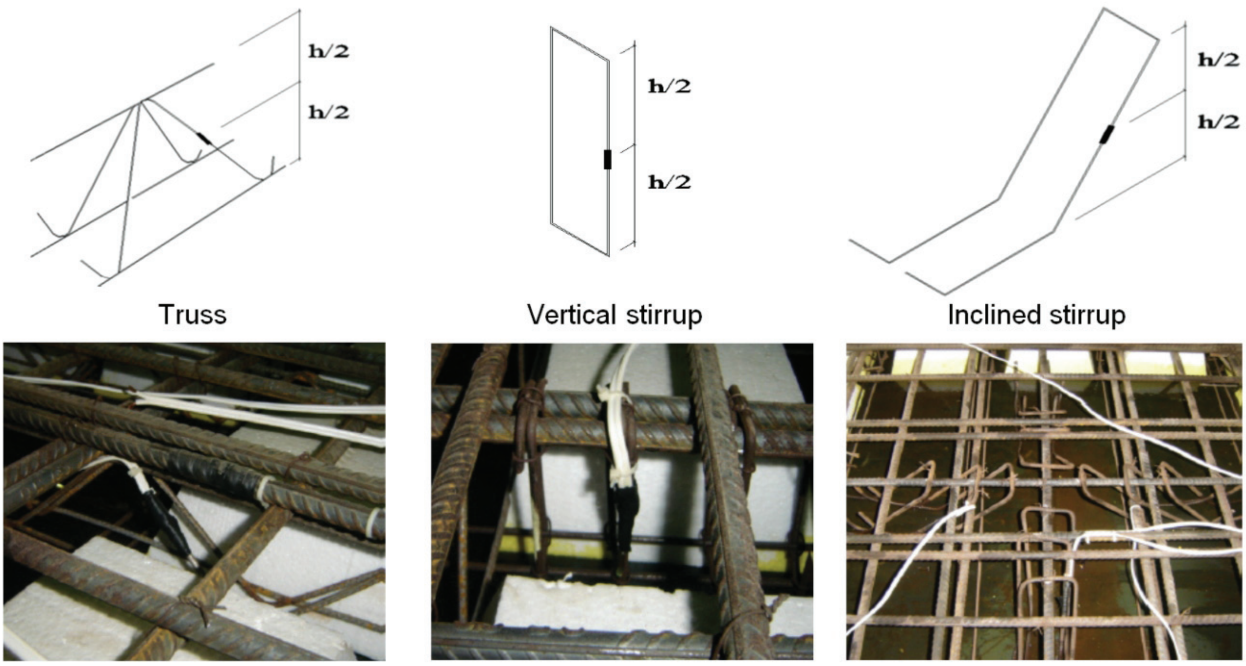


Figure 9 – Testing system

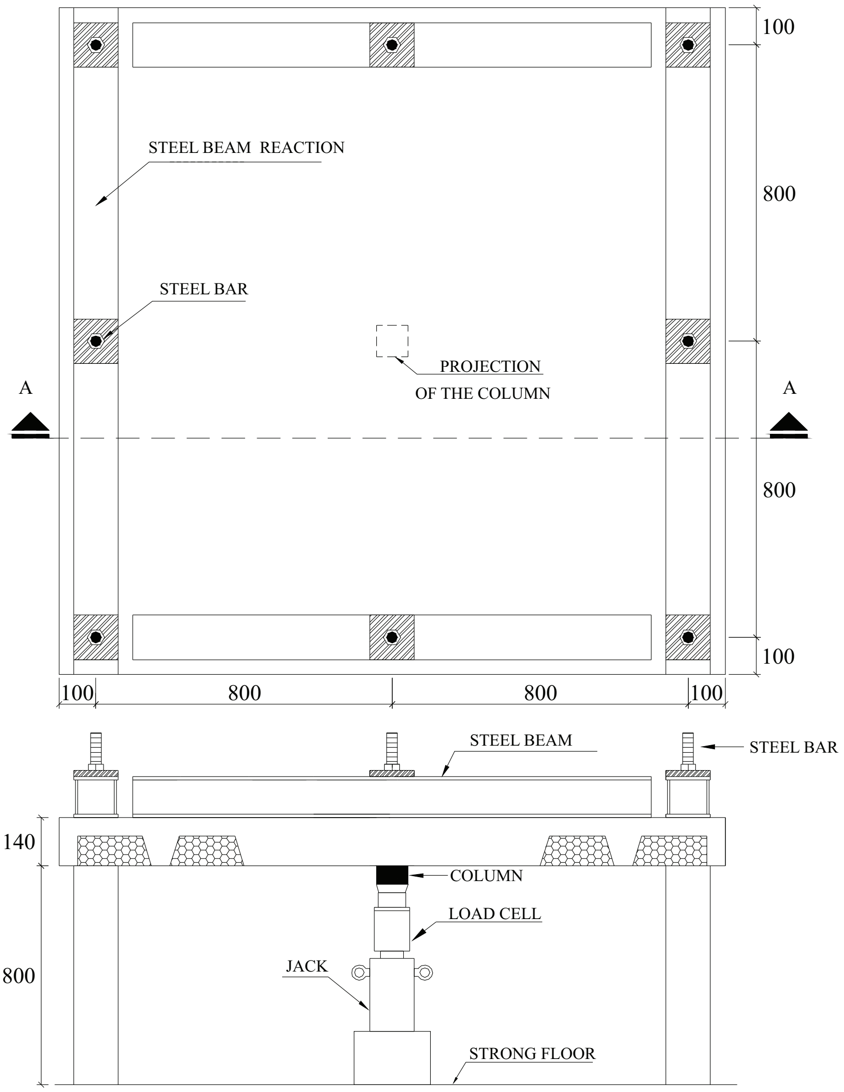


Figure 10 – Details of the loading system

Table 2 – Concrete mechanical properties

Slab	Age (day)	f_c (MPa)	f_t (MPa)	E_c (GPa)
L1	54	37	3.5	24.9
L2	49	38	3.4	17.8
L3	64	41	2.5	17.9
L4	60	39	2.9	18.7
L5	56	38	2.8	26.5
L6	70	40	2.1	27.0
L7	68	41	1.8	27.8
L8	74	39	1.8	27.6

f_t:tensile strength of concrete;
E_c: modulus of elasticity of concrete.

Table 3 – Steels mechanical properties

ϕ (mm)	f_{ys} (MPa)	f_u (MPa)	ε_{ys} (‰)	E_s (GPa)
4.2	630	705	4.7	233
6.0	590	688	4.5	236
6.3	588	794	4.4	242
12.5	601	740	2.5	255

ϕ: bars diameter;
f_{ys} and f_u: yield and failure strength, respectively;
ε_{ys}: yield strain;
E_s: longitudinal modulus of elasticity.

slabs L6 and L7 were classified as flexure, followed by punching (punching cone formation). The remaining slabs failure by punching, characterized by a sudden and brittle failure. Figure 17 shows the failure surface of the slabs and Figure 18 shows the cracking pattern of the slabs at the end of the tests.

3.7 Comparisons between experimental and estimates results

Table 5 presents the estimated failure loads according to NBR 6118:2003 for punching and shear in the ribs, and flexural resistance from yield line theory using the equations developed by Oliveira (2003) [10], and the experimental failure loads. In general, the normative estimates for shear strength of ribbed slabs were conservatives, including those with shear reinforcement in the ribs, showing that the security of the slabs would be even greater if the failures were in the ribs.
Regarding the punching resistance, the estimates from NBR 6118:2003, showed that this code tends to overestimate the results, and in some cases considered satisfactory, although not always coincide with the observed failure mode, but also provided some conservative results. For the flexural strength can be observed that with the exception of L6 slab, all slabs had higher estimates to the experimental results. In the slabs without punching reinforcement the resistance was overestimated, on average, 23%, indicating that a flexural failure was far, with the slabs L1 and L2 presenting punching failures. The slabs with punching reinforcement L6 and L7 showed estimated resistances close to the experimental ones and were satisfactory even for the observed failure modes.

4. Conclusions

The slabs with shear reinforcement in the ribs (L2, L3, L4 and L5) did not achieve significant resistance in relation to the reference slab (L1). In relation to the slabs with punching reinforcement (L6, L7 and L8), they showed superior resistance to the slab L1, around 26%, confirming the efficiency of the inclined stirrups as punching reinforcement.
Considering the slab (L1) the ultimate strength was too underestimated for a failure in the ribs. The other slabs also had estimates well below the experimental results. For the slabs with shear and punching reinforcements (L6, L7 and L8) the disparity was much greater, resulting in estimated failure loads of approximately 2 times smaller than the experimental ones, since the contribution of the punching reinforcement is not considered in this design.
In to the punching failure loads estimates, most of the results were satisfactory, with differences between the experimental results up to 5%. It must be observed the not ever a punching failure was observed, even for slabs in which the results were considered satisfactory.

5. Acknowledgements

The authors thank CNPq, CAPES, FADESP, ITEGAM and FAPESP for financial support for this work.

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Figure 11 – Slabs vertical displacements versus applied loadings

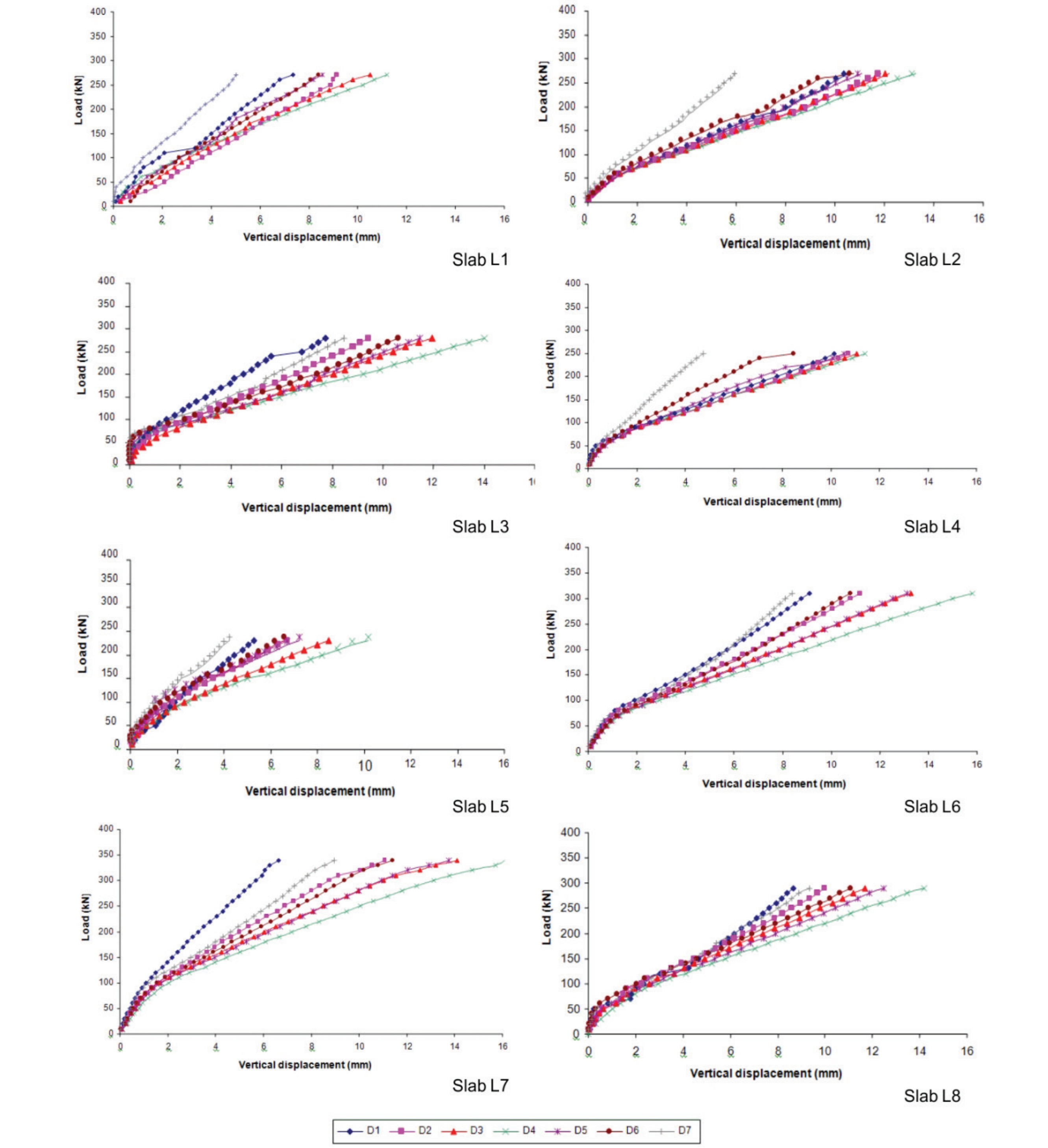

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Figure 12 – Slabs middle span vertical displacements (D4)

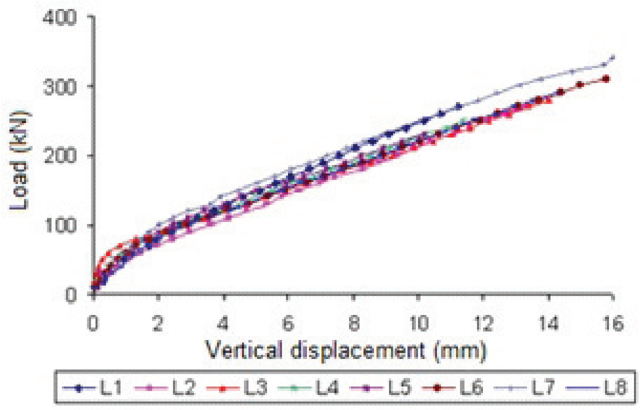

Figure 13 – Slabs concrete strains

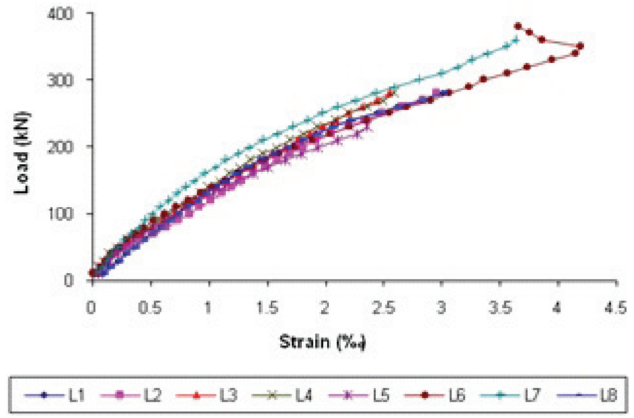

Figure 14 – Slabs flexural reinforcement strains

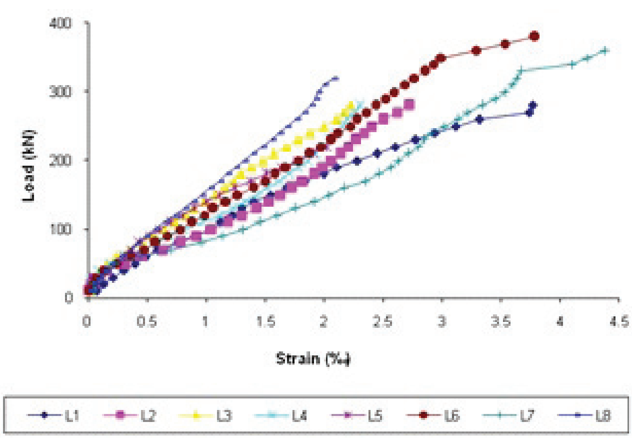

Figure 15 – Slabs L2 and L5 shear reinforcement in the ribs strains

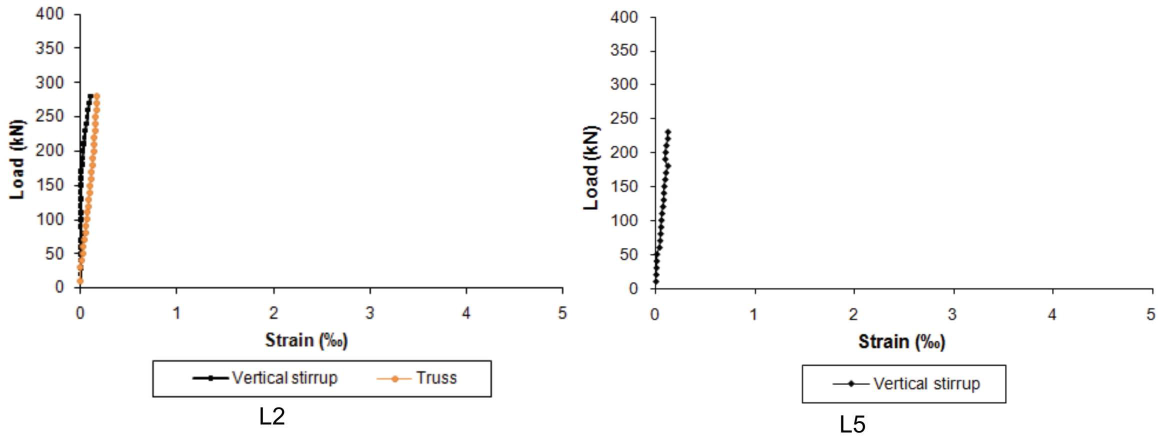

Figure 16 – Slabs L6 and L8 shear reinforcement in the solid region strains

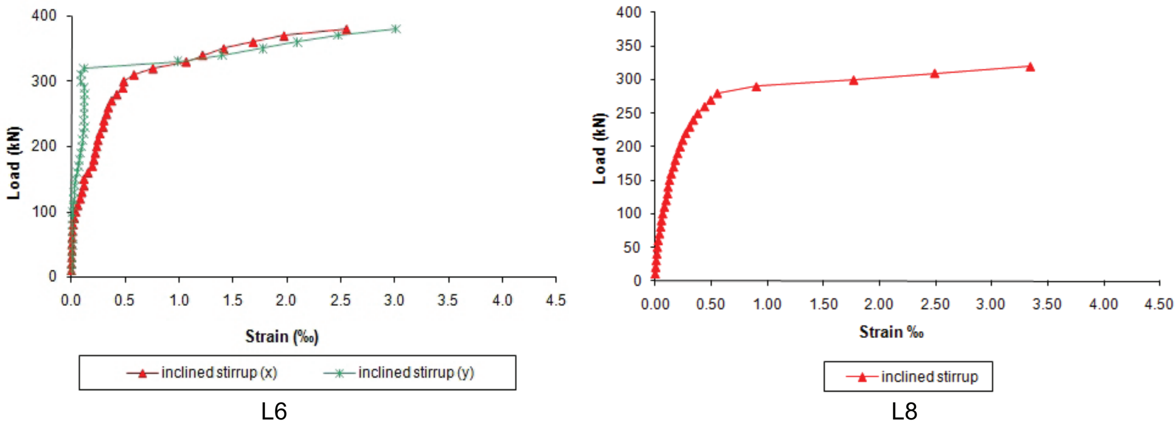

Table 4 – Slabs loads and modes of failure

Slab	d (mm)	ρ	f_c (MPa)	$\varepsilon_{s,max}$ (‰)	$\varepsilon_{c,max}$ (‰)	Shear reinforcement		P_u (kN)	Failure mode
						Rib	Punching		
L1	120	0.0127	41	3.8	3.1	-	-	280.0	punch
L2	106	0.0144	37	2.7	2.9	Truss	-	278.5	punch
L3	111	0.0137	38	2.2	2.8	Truss	-	287.5	punch
L4	118	0.0129	39	2.3	2.8	Truss	-	287.0	punch
L5	115	0.0133	38	2.8	2.5	Vertical stirrup	-	235.0	punch
L6	104	0.0147	40	3.8	4.2	Truss	Inclined stirrup	380.0	flexure
L7	112	0.0136	41	4.4	3.6	Vertical stirrup	Inclined stirrup	361.0	flexure
L8	108	0.0141	39	2.2	-	Inclined stirrup	Inclined stirrup	322.0	punch

$\varepsilon_{s,max}$: maximum observed tensile strain for flexural reinforcement;
$\varepsilon_{c,max}$: maximum observed compressive strain for bottom concrete surface;
P_u: experimental failure load.

Figure 17 – Slabs failure surface in the solid region

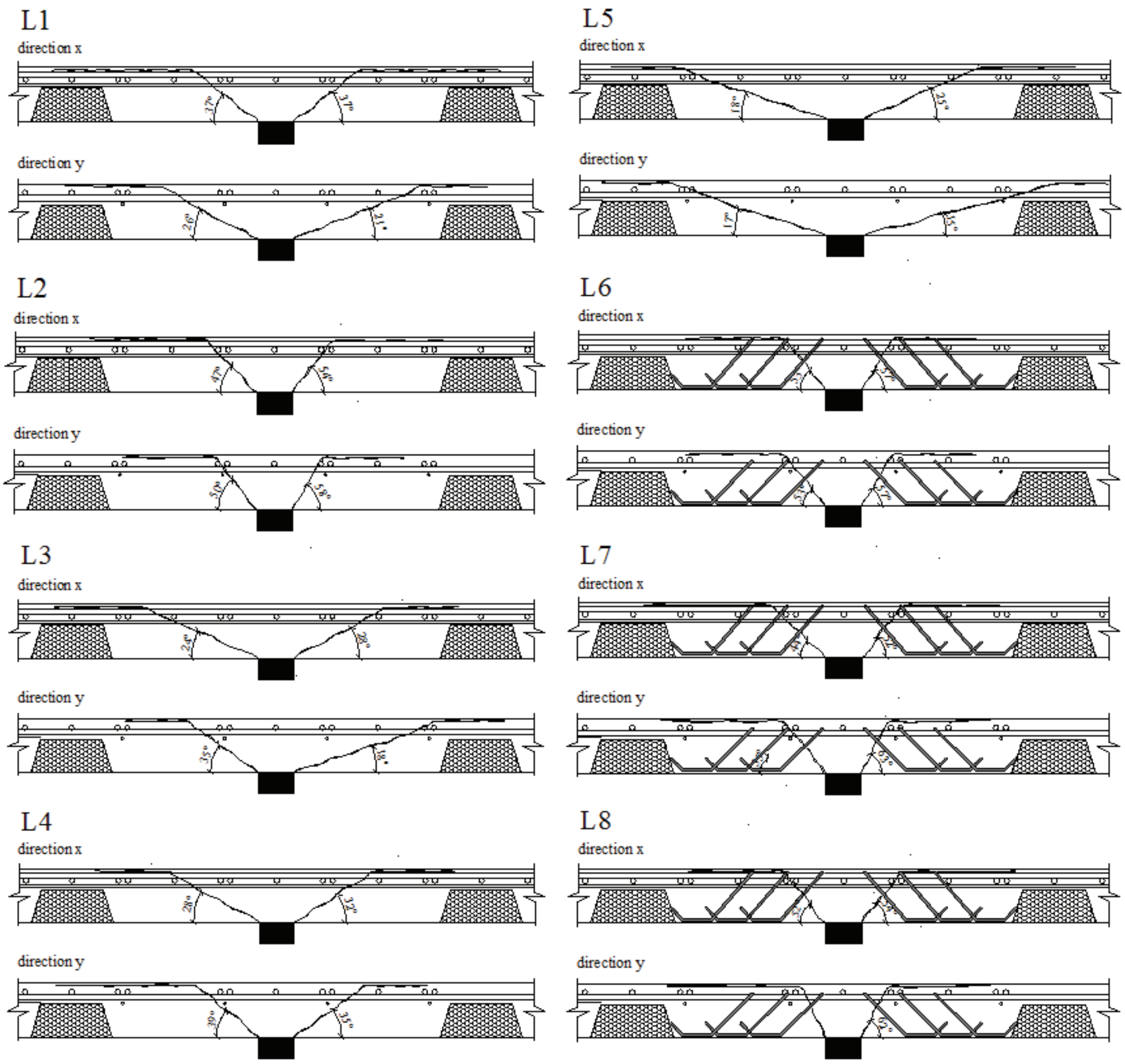

Figure 18 – Cracking patterns of the slabs

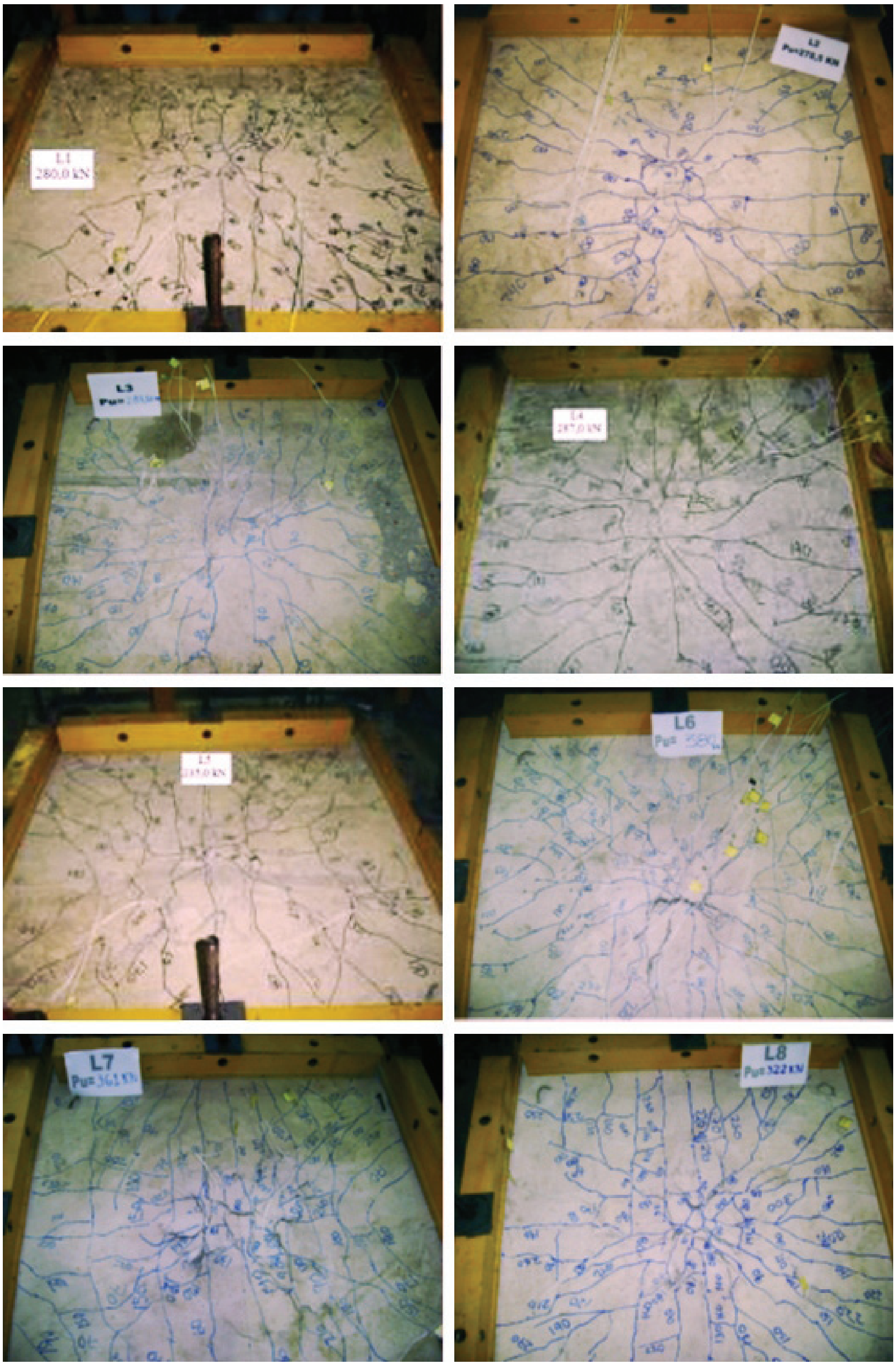

Table 5 – Estimated and experimental failure loads

Slab	d (mm)	ρ	f_c (MPa)	P_{punch} (kN)	P_{shear} (kN)	$P_{flexure}$ (kN)	P_u (kN)	Failure Mode
L1	120	0.0127	41	345	144	363	280.0	punch
L2	106	0.0144	37	288	257	358	278.5	punch
L3	111	0.0137	38	307	271	359	287.5	punch
L4	118	0.0129	39	333	291	361	287.0	punch
L5	115	0.0133	38	320	228	359	235.0	punch
L6	104	0.0147	40	431	258	362	380.0	flexure
L7	112	0.0136	41	477	229	363	361.0	flexure
L8	108	0.0141	39	448	266	361	322.0	punch

P_{punch}: estimated punching failure load; P_{shear}: estimated shearing failure load for ribs; $P_{flexure}$: estimated flexural failure load.