INTEGRAL TRANSFORM SOLUTION FOR THE FORCED CONVECTION OF HERSCHEL-BULKLEY FLUIDS IN CIRCULAR TUBES AND PARALLEL-PLATES DUCTS

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Abstract - The thermal entry region in laminar forced convection of Herschel-Bulkley fluids is solved analytically through the integral transform technique, for both circular and parallel-plates ducts, which are maintained at a prescribed wall temperature or at a prescribed wall heat flux. The local Nusselt numbers are obtained with high accuracy in both developing and fully-developed thermal regions, and critical comparisons with previously reported numerical results are performed.
Keywords: Integral transform technique, internal forced convection, Herschel-Bulkley fluids.

INTRODUCTION

The analysis of convection heat transfer to non-newtonian fluids inside ducts is of great importance in the design of thermal equipment in the pharmaceutical, food and petrochemical industries. This non-newtonian behavior is encountered in materials such as: solutions or melts of polymeric materials, oils and greases, cosmetics, toothpaste, soap and detergents, paints, cement and drilling muds. During the drilling operation, particularly in the petroleum industry, certain drilling muds and cements are of fundamental importance in providing a good operation and the well cementing. These drilling muds follow a non-newtonian behavior based on the Herschel-Bulkley model, a power-law fluid model with yield-stress, and their rheological properties are sensitive to the temperature. Therefore, the determination of the temperature distributions in such fluids are necessary for the complete control of these properties during the drilling operation.

Thus, in this context, the present work deals with the thermal problem in both the entry and fully-developed regions of fluids that follow the Herschel-Bulkley model, inside circular and parallel-plates ducts. The solution of this problem is obtained by applying the integral transform technique.

The analytic solution in the thermal entry region for both laminar and turbulent forced convection inside ducts involves difficulties due to the related auxiliary eigenvalue problem (Özisik et al., 1989). Previous works have employed purely numerical techniques to solve the eigenvalue problem (Sellars et al., 1956; Shibani and Özisik, 1977), such as the Runge-Kutta method, but only the first few eigenvalues can be determined by this technique. Consequently, it is not feasible to calculate heat transfer results in regions which are very close to the duct inlet because a large number of eigenvalues are needed for the computation of the series expansion based on the eigenfunctions.

Alternatively, numerical methods as finite-difference schemes have been employed to solve the complete system of partial differential equations, but accurate results were not obtained by these numerical schemes (Forrest and Wilkinson, 1973; Lin and Shah, 1978; Nouar et al., 1994; Mendes and Naccache, 1995).

In the 80□s, Mikhailov and Vulchanov (1983) and Mikhailov and Özisik (1984) advanced the so-called Sign-Count Method based on the approaches of Wittrick and Williams (1971, 1974), in order to solve the related Sturm-Liouville type eigenvalue problem, which permits the automatic and highly accurately determination of as many eigenvalues and eigenfunctions as are needed.

More recently, Cotta (1993) and Mikhailov and Cotta (1994), also developed an approach to solve eigenvalue problems based on the ideas of Generalized Integral Transform Technique (GITT) which was demonstrated to be as efficient and safe as the sign-count method.

Therefore, to alleviate such difficulties pointed out in the solution of this heat transfer problem, and to be able to perform heat transfer calculations in regions very close to the duct inlet with a high degree of accuracy, the ideas on Integral Transform Technique in conjunction with the well-established sign-count method and GITT are used in the present work, aimed at establishing benchmark results for this problem.

ANALYSIS

The rheological behavior of the non-newtonian fluid described here is given by the Herschel-Bulkley model in the following form:

$$\tau = \tau_y + K\dot{\gamma}^n \; ; \quad \tau > \tau_y \quad (1.a)$$

$$\dot{\gamma}^n = 0 \qquad ; \quad \tau \le \tau_y \quad (1.b)$$

where, τ is the shear stress, τ_y is the yield stress, $\dot{\gamma}$ is the shear rate, K is the consistency index of the fluid and n is the power-law exponent which is less than unity for pseudoplastic fluids (shear thinning materials) and greater than unity for dilatant fluids (shear thickening materials).

For the fully-developed region of a circular or a parallel-plates duct, the momentum equation in the axial coordinate z, is simplified to yield:

$$\frac{1}{r^p}\frac{d}{dr}\left(r^p \tau\right) = \left(-\frac{dp}{dz}\right), \quad (2.a)$$

in $0 < r < b$, $z > 0$

subjected to the following boundary conditions

$$\frac{du}{dr} = 0 \quad , \quad \text{at} \quad r = 0 \quad (2.b)$$

$$u = 0 \quad , \quad \text{at} \quad r = b \quad (2.c)$$

where the exponent p denotes the channel geometry and is written as:

$$p = \begin{cases} 0, \text{for parallel -plates channel} \\ 1, \text{for circular tubes} \end{cases} \quad (3.a, b)$$

Then, introducing equations (1) in equation (2.a), noting that $\dot{\gamma} = -du/dr$ and after the integrations are performed, the fully-developed velocity profile for Herschel-Bulkley fluids is given by:

$$u(r) = \frac{n\left(-\frac{dp}{dz}\right)^{1/n} b^{(n+1)/n}}{(n+1)(p+1)^{1/n} K^{1/n}} \left(1 - \frac{b_o}{b}\right)^{(n+1)/n},$$

$$\text{for } r \leq b_o \qquad (4.a)$$

and,

$$u(r) = \frac{n\left(-\frac{dp}{dz}\right)^{1/n} b^{(n+1)/n}}{(n+1)(p+1)^{1/n} K^{1/n}} \left\{ \left[1 - \frac{\tau_y (p+1)}{\left(-\frac{dp}{dz}\right) b}\right]^{(n+1)/n} - \left[\frac{r}{b} - \frac{\tau_y (p+1)}{\left(-\frac{dp}{dz}\right) b}\right]^{(n+1)/n} \right\}, \quad \text{for } r \geq b_o \qquad (4.b)$$

where b_o represents the radius of the plug-flow region defined as:

$$b_o = \frac{(p+1)\,\tau_y}{\left(-\frac{dp}{dz}\right)} \qquad (5)$$

The velocity profile given by equations (4) for a Herschel-Bulkley fluid is split in two distinct regions, one for $r \leq b_o$ which denotes the plug-flow region where $\tau \leq \tau_y$, and the fluid behaves like a solid plug, and another region for $r \geq b_o$ where $\tau > \tau_y$, and refers to that part of the fluid which is in shear flow (Forrest and Wilkinson, 1973).

In the analysis of the thermal problem, we consider steady-state laminar forced convection heat transfer to hydrodynamically developed flow in the thermal entry region of an incompressible non-newtonian fluid that follows the Herschel-Bulkley model, described by equations (1) and (4), inside both circular and parallel-plates ducts maintained at a prescribed wall temperature T_w, or at a prescribed wall heat flux q_w. The fluid enters the channels with a constant uniform temperature T_o.

Axial heat conduction and viscous dissipation are neglected and the physical properties are considered temperature independent.

Then, the mathematical formulation of this heat transfer problem in dimensionless form is defined by:

$$W(R)\ \frac{\partial \theta\ (R, Z)}{\partial Z} = \frac{\partial}{\partial R}\left[R^{p}\ \ \frac{\partial \theta\ (R, Z)}{\partial R}\right],$$

$$\text{in} \quad 0 < R < 1, Z > 0 \qquad (6.a)$$

subjected to the boundary and inlet conditions:

$$\left.\frac{\partial \theta}{\partial R}\right|_{R=0} = 0 \quad ;$$

$$(1-m)\ \theta\,(1,Z) + m\left.\frac{\partial \theta}{\partial R}\right|_{R=1} = 1\ , \qquad Z > 0 \qquad (6.b, c)$$

$$\theta\,(R,0) = 0 \qquad , \quad 0 \leq R \leq 1 \qquad (6.d)$$

where in the boundary conditions (6.c), the coefficient m identifies whether the duct wall is subjected to a prescribed temperature or to a prescribed heat flux, in the following form:

$$m = \begin{cases} 0\ , \text{ for prescribed wall temperatu re} \\ 1\ , \text{ for prescribed wall heat flux} \end{cases} \qquad (7.a, b)$$

In equations (6) above the following dimensionless groups were used:

$$R = \frac{r}{b} \ ; \ Z = \frac{z}{D_h\ Re\ Pr} \ ; \ Re = \frac{\rho\, u_m^{2-n}\ D_h^{n}}{K} \ ;$$

$$Pr = \frac{K}{\rho\, u_m^{1-n}\ D_h^{n-1}\ \alpha} \ ; \ U\,(R) = \frac{u\,(r)}{u_m} \ ; \ W\,(R) = \frac{R^{p}}{2^{4-2p}}\, U\,(R) \ ;$$

$$\theta\,(R, Z) = \frac{T\,(r,z)\ - T_o}{(1-m)\ (T_w - T_o) + m\left(\dfrac{q_w\, b}{k}\right)} \qquad (8.a\text{-}g)$$

where D_h is the hydraulic diameter, defined as:

$$D_h = 2^{2-p}\ b \qquad (9)$$

The velocity profile given by equations (4) is written in dimensionless form by introducing the groups (8), or:

$$U(R) = \frac{n(p+1)}{2(n+1)} \frac{1}{f\,Re} \left[\frac{f\,Re}{(p+1)\,2^{1-p}} - Y \right]^{(n+1)/n} ,$$
$$\text{for } R \le R_o \qquad (10.a)$$

and,

$$U(R) = \frac{n(p+1)}{2(n+1)} \frac{1}{f\,Re} \left\{ \left[\frac{f\,Re}{(p+1)\,2^{1-p}} - Y \right]^{(n+1)/n} - \left[\frac{R\,f\,Re}{(p+1)\,2^{1-p}} - Y \right]^{(n+1)/n} \right\}, \text{ for } R \ge R_o \qquad (10.b)$$

where the additional groups employed in the above equations are R_o (the dimensionless radius of the plug-flow region), Y (the yield number) and f (the Fanning friction factor), i. e.,

$$R_o = \frac{b_o}{b} \; ; \; Y = \frac{\tau_y}{K} \left(\frac{D_h}{u_m} \right)^n \; ;$$
$$f = \left(-\frac{dp}{dz} \right) \frac{D_h}{2 \rho u_m^2} \qquad (11.a\text{-}c)$$

The Fanning friction factor and the dimensionless radius of the plug-flow region are functions of the Reynolds and yield numbers. The determination of both quantities is obtained by resolution of a simple transcendental equation, and due to space limitations is not described here. Table 1 below shows some results for the product f Re.

The problem given by equations (6) can be solved by the classical integral transform technique (Mikhailov and Özisik, 1984; Cotta, 1993). In order to make the boundary conditions (6.c) homogeneous, so as to obtain a better computational performance in the series expansion, a splitting-up procedure is proposed as (Mikhailov, 1977; Mikhailov and Özisik, 1984):

$$\theta(R,Z) = m\,\theta_{av}(Z) + \theta_p(R) + \theta_h(R,Z) \qquad (12)$$

$\theta_{av}(Z)$ is the average temperature, defined as:

$$\theta_{av}(Z) = \frac{\int_0^1 W(R)\,\theta(R,Z)\,dR}{\int_0^1 W(R)\,dR} \qquad (13)$$

and, for the case of a prescribed wall heat flux, when we have all boundary conditions of the second kind, the average temperature is given *a priori* in the form:

$$\theta_{av}(Z) = \frac{Z}{\int_0^1 W(R)\,dR} \qquad (14)$$

Table 1: Product f Re computed from the present analysis

Yield	Parallel–Plates Channel				Circular Tube			
number	f Re				f Re			
Y	n = 0.5	n = 0.75	n = 1	n = 1.5	n = 0.5	n = 0.75	n = 1	n = 1.5
0	8.00000	13.9552	24.0000	69.6745	6.32455	10.1023	16.0000	39.7175
1	10.6139	16.7855	26.9945	72.8860	8.79215	12.6895	18.6659	42.4731
5	20.3899	27.5582	38.6656	85.8155	18.1880	22.7189	29.1995	53.5825
10	31.9097	40.2208	52.5515	101.836	29.3964	34.7020	41.9439	67.4775
20	54.0525	64.2640	78.8535	132.929	51.1030	57.7230	66.4215	94.7670

Now, introducing equation (12) into equations (6), the following problems for the potentials $\theta_p(R)$ and $\theta_h(R,Z)$ are obtained:

$$\frac{d}{dR}\left[R^p \frac{d\theta_p(R)}{dR}\right] = m\,W(R)\,\frac{d\theta_{av}(R)}{dZ} \quad ,$$
$$\text{in} \quad 0 < R < 1 \qquad (15.a)$$

$$\left.\frac{d\theta_p(R)}{dR}\right|_{R=0} = 0 \quad ;$$
$$(1-m)\,\theta_p(1) + m \left.\frac{d\theta_p(R)}{dR}\right|_{R=1} = 1 \qquad (15.b,\ c)$$

with its respective solution:

$$\theta_p(R) = (1-m) + m\left\{\int_1^R \frac{\left[\int_0^{R'} W(R'')\,dR''\right]}{R'^p\left[\int_0^1 W(R)\,dR\right]}\,dR' - \int_0^1 W(R)\left[\int_1^R \frac{\left[\int_0^{R'} W(R'')\,dR''\right]}{R'^p\left[\int_0^1 W(R)\,dR\right]^2}\,dR'\right]dR\right\} \quad (16)$$

and,

$$W(R)\frac{\partial\theta_h(R,Z)}{\partial Z} = \frac{\partial}{\partial R}\left[R^p\,\frac{\partial\theta_h(R,Z)}{\partial R}\right],$$
$$\text{in} \quad 0 < R < 1, Z > 0 \quad (17.a)$$

with initial and boundary conditions:

$$\theta_h(R,0) = -\theta_p(R), \qquad 0 \le R \le 1 \quad (17.b)$$

$$\left.\frac{\partial\theta_h}{\partial R}\right|_{R=0} = 0 \quad ;$$

$$(1-m)\,\theta_h(1,Z) + m\left.\frac{\partial\theta_h}{\partial R}\right|_{R=1} = 0\ ,$$
$$Z > 0 \quad (17.c, d)$$

The homogeneous problem defined above by equations (17) can also be solved by the classical integral transform technique. Then, following the procedures of this technique, the appropriate eigenvalue problem needed for its solution is given by:

$$\frac{d}{dR}\left[R^p\,\frac{d\psi_i(R)}{dR}\right] + \mu_i^2\,W(R)\,\psi_i(R) = 0,$$
$$\text{in} \quad 0 < R < 1 \quad (18.a)$$

$$\left.\frac{d\psi_i(R)}{dR}\right|_{R=0} = 0 \quad ;$$

$$(1-m)\,\psi_i(1) + m\left.\frac{d\psi_i(R)}{dR}\right|_{R=1} = 0 \quad (18.b,c)$$

where $\psi_i(R)$ and μ_i are, respectively, the eigenfunctions and eigenvalues. The eigenvalue problem allows for the development of the

following integral transform pair:

$$\underbrace{\theta_h(R,Z) = \sum_{i=1}^{\infty} \frac{1}{N_i^{1/2}} \psi_i(R)\, \bar{\theta}_i(Z)}_{\text{inversion}} \quad , (19.a,b)$$

$$\underbrace{\bar{\theta}_i(Z) = \frac{1}{N_i^{1/2}} \int_0^1 W(R)\, \psi_i(R)\, \theta_h(R,Z)\, dR}_{\text{transform}}$$

where the normalization integral is given by:

$$N_i = \int_0^1 W(R)\, \psi_i^2(R)\, dR \quad (20)$$

Then, taking the integral transform of the system given by equation (17), these equations are operated with $\frac{1}{N_i^{1/2}} \int_0^1 \psi_i(R) dR$, and we obtain the following ordinary differential equation for the transformed potential, $\bar{\theta}_i(Z)$:

$$\frac{d\bar{\theta}_i(Z)}{dZ} + \mu_i^2\, \bar{\theta}_i(Z) = 0 \quad (21.a)$$

with the transformed inlet condition given by:

$$\bar{\theta}_i(0) = \bar{f}_i = -\frac{1}{N_i^{1/2}} \int_0^1 W(R)\, \psi_i(R)\, \theta_p(R)\, dR \quad (21.b)$$

The solution for the transformed potential given by equations (21) is readily obtained in the form:

$$\bar{\theta}_i(Z) = \bar{f}_i \exp(-\mu_i^2 Z) \quad (22)$$

Therefore, introducing equation (22) into the inversion formula (19.a), the solution for $\theta_h(R,Z)$ is determined as:

$$\theta_h(R,Z) = \sum_{i=1}^{\infty} \frac{\bar{f}_i}{N_i^{1/2}} \psi_i(R) \exp(-\mu_i^2 Z) \qquad (23)$$

Thus, equation(23) in conjunction with equation (16) for $\theta_p(R)$ complete the solution for the potential $\theta(R,Z)$ defined in equation (12). For the case of a prescribed wall heat flux, the average temperature is given by equation (14). For the case

of a prescribed wall temperature, when it isn□t determined *a priori*, it may be readily obtained by substituting the solution for $\theta(R,Z)$, equation (12), with m = 0, into equation (13) to yield:

$$\theta_{av}(Z) = 1 - \sum_{i=1}^{\infty} \frac{\bar{f}_i^2}{\left(\int_0^1 W(R)\, dR\right)} \exp(-\mu_i^2 Z) \qquad (24)$$

The local Nusselt number for both situations is defined as:

$$Nu(Z) = \frac{h(z)\, D_h}{k} = \frac{2^{2-p} \left.\frac{\partial\theta}{\partial R}\right|_{R=1}}{\theta(1,Z) - \theta_{av}(Z)} \qquad (25)$$

after substituting equation (12) for $\theta(1,Z)$, and equation (24), for the case of a prescribed wall temperature, into equation (25) above, the two distinct situations result:

$$Nu(Z) = \frac{\sum_{i=1}^{\infty} \mu_i^2\, \bar{f}_i^2 \exp(-\mu_i^2 Z)}{(p+1)\, 2^{2-p} \sum_{i=1}^{\infty} \bar{f}_i^2 \exp(-\mu_i^2 Z)}, \qquad (26)$$

for m = 0

the asymptotic Nusselt number, Nu_∞ ,for this case, is obtained from equation (26), by considering only the first term in the summation, to yield:

$$Nu_\infty = \frac{\mu_1^2}{(p+1)\, 2^{2-p}} \qquad (27)$$

while,

$$Nu(Z) = \frac{2^{2-p}}{\theta_p(1) + \sum_{i=1}^{\infty} \frac{\bar{f}_i}{N_i^{1/2}} \psi_i(1) \exp(-\mu_i^2 Z)}, \quad (28)$$

for m = 1

where, $\theta_p(1)$ is obtained from equation (16) in the form

$$\theta_p(1) = -\int_0^1 W(R) \left[\int_1^R \frac{\left[\int_0^{R'} W(R'')\, dR'' \right]}{R'^p \left[\int_0^1 W(R)\, dR \right]^2} dR' \right] dR \Bigg\} \quad (29)$$

and in this case the asymptotic Nusselt number, Nu_∞, is given by equation (28), making $Z \to \infty$, so that

$$Nu_\infty = \frac{2^{2-p}}{\theta_p(1)} \quad (30)$$

To complete the solution it is necessary to evaluate the eigenvalues, μ_i, the eigenfunctions $\psi_i(R)$ and the normalization integral N_i of the eigenvalue problem (18). Here, for instance, we have used both the sign-count method established in references (Mikhailov and Vulchanov, 1983; Mikhailov and Özisik, 1984) and the generalized integral transform technique (Cotta, 1993; Mikhailov and Cotta, 1994) to determine the eigenvalues and other related eigenquantities necessary to compute the average temperature and the local Nusselt numbers from equations (24) and (26-30).

RESULTS AND DISCUSSION

First, the eigenvalues and other related eigenquantities were obtained by the two approaches cited above. The results obtained through the two approaches are in perfect agreement. Due to space limitations they are not listed here. Then, the average temperature, $\theta_{av}(Z)$, and the local Nusselt numbers, $Nu(Z)$, were calculated.

In Tables 2.a to 2.d, the present results are validated against previous results for power-law fluids presented by Cotta and Özisik (1986a, 1986b), in the thermal entry region, for the case of Y = 0 and n = 1/3, 1 and 3. From these tables it can be noticed that the results are in excellent agreement, providing a direct validation of the numerical code developed in this work.

Table 3.a shows the asymptotic Nusselt number from the present analysis and its comparison with those computed by Lin and Shah (1978) for various power-law exponents and yield numbers for both circular tubes and parallel-plates channels and considering a prescribed wall temperature. A similar analysis is also shown in Table 3.b for the case of a prescribed wall heat flux. An excellent agreement between the results can be observed from these tables.

Table 2.a: Local Nusselt numbers for parallel-plates channel (prescribed wall heat flux)

Z	Nu(Z)					
	n = 1/3		N = 1		n = 3	
0.000001	175.73+	175.77*	148.78+	148.78*	137.01+	137.02*
0.000005	102.47	102.48	86.955	86.954	80.162	80.165
0.000010	81.207	81.216	69.011	69.011	63.660	63.662
0.000050	47.335	47.338	40.420	40.420	37.361	37.361
0.000100	37.549	37.551	32.156	32.156	29.758	29.758
0.000500	22.115	22.115	19.113	19.112	17.753	17.753
0.001000	17.757	17.757	15.427	15.427	14.360	14.360
0.005000	11.311	11.311	9.9878	9.9878	9.3569	9.3569
0.010000	9.8802	9.8803	8.8032	8.8031	8.2774	8.2774
0.050000	9.1488	9.1489	8.2355	8.2355	7.7772	7.7773
0.100000	9.1484	9.1485	8.2353	8.2353	7.7771	7.7771
0.200000	9.1484	9.1485	8.2353	8.2353	7.7771	7.7771

Table 2.b: Local Nusselt numbers for circular tube (prescribed wall heat flux)

Z	Nu(Z)					
	n = 1/3		n = 1		n = 3	
0.000001	147.69+	147.72*	129.18+	129.21*	121.61+	121.64*
0.000005	85.882	85.891	75.180	75.191	70.796	70.806
0.000010	67.940	67.946	59.504	59.510	56.043	56.049
0.000050	39.341	39.343	34.508	34.511	32.516	32.518

0.000100	31.068	31.069	27.274	27.276	25.706	25.707
0.000500	17.973	17.974	15.812	15.813	14.911	14.911
0.001000	14.239	14.239	12.538	12.538	11.824	11.824
0.005000	8.5065	8.5066	7.4936	7.4937	7.0609	7.0610
0.010000	6.9893	6.9893	6.1481	6.1481	5.7849	5.7850
0.050000	5.1947	5.1948	4.5138	4.5139	4.2098	4.2098
0.100000	5.0613	5.0613	4.3748	4.3748	4.0640	4.0640
0.200000	5.0526	5.0527	4.3637	4.3637	4.0507	4.0507

+ - Present work
* - Cotta and Ozisik (1986a)

Table 2.c: Local Nusselt numbers for parallel-plates channel (prescribed wall temperature)

Z	Nu(Z)					
	n = 1/3		n = 1		n = 3	
0.000001	145.06+	145.06*	122.94+	122.94*	113.29+	113.29*
0.000005	84.465	84.466	71.830	71.830	66.284	66.284
0.000010	66.887	66.887	56.999	56.999	52.643	52.643
0.000050	38.901	38.901	33.379	33.379	30.915	30.915
0.000100	30.826	30.827	26.560	26.560	24.642	24.642
0.000500	18.126	18.127	15.830	15.830	14.767	14.767
0.001000	14.566	14.566	12.822	12.822	11.999	11.999
0.005000	9.4438	9.4439	8.5166	8.5166	8.0479	8.0479
0.010000	8.4902	8.4902	7.7405	7.7405	7.3495	7.3495
0.050000	8.2274	8.2275	7.5407	7.5407	7.1776	7.1776
0.100000	8.2274	8.2275	7.5407	7.5407	7.1776	7.1776
0.200000	8.2274	8.2275	7.5407	7.5407	7.1776	7.1776

Table 2.d: Local Nusselt numbers for circular tube (prescribed wall temperature)

Z	Nu(Z)					
	n = 1/3		n = 1		n = 3	
0.000001	121.74+	121.74*	106.54+	106.54*	100.31+	100.31*
0.000005	70.623	70.623	61.877	61.876	58.284	58.284

0.000010	55.789	55.788	48.914	48.913	46.084	46.084
0.000050	32.153	32.153	28.254	28.254	26.638	26.638
0.000100	25.321	25.321	22.279	22.279	21.012	21.012
0.000500	14.523	14.523	12.824	12.824	12.107	12.107
0.001000	11.452	11.452	10.130	10.130	9.5670	9.5670
0.005000	6.7613	6.7613	6.0015	6.0015	5.6673	5.6673
0.010000	5.5375	5.5375	4.9161	4.9161	4.6372	4.6372
0.050000	4.2223	4.2223	3.7100	3.7100	3.4678	3.4678
0.100000	4.1762	4.1762	3.6581	3.6581	3.4107	3.4107
0.200000	4.1753	4.1753	3.6568	3.6568	3.4090	3.4090

+ - Present work
* - Cotta and Özisik (1986b)

Table 3.a: Asymptotic Nusselt numbers for the case of prescribed wall temperature

Parallel-Plates Channel								
Y	**Nu_∞**							
	n = 0.5		**n = 0.75**		**n = 1**		**n = 1.5**	
0	7.9398+	7.940*	7.6897+	7.690*	7.5407+	7.541*	7.3714+	7.372*
1	8.1813	8.182	7.8299	7.830	7.6182	7.619	7.3941	7.395
5	8.7016	8.701	8.2281	8.228	7.8833	7.884	7.4872	7.488
10	8.9859	8.985	8.5102	8.511	8.1150	8.115	7.5957	7.597
20	9.2409	9.239	8.8101	8.810	8.4018	8.401	7.7737	7.774
Circular Tube								
Y	**Nu_∞**							
	n = 0.5		**n = 0.75**		**n = 1**		**n = 1.5**	
0	3.9494+	3.947*	3.7634+	3.760*	3.6568+	3.654*	3.5392+	3.536*
1	4.1707	4.168	3.8914	3.888	3.7287	3.725	3.5619	3.559
5	4.6722	4.668	4.2757	4.274	3.9899	3.987	3.6589	3.656
10	4.9474	4.944	4.5487	4.545	4.2209	4.218	3.7773	3.775
20	5.1921	5.189	4.8326	4.828	4.4986	4.495	3.9695	3.967

+ - Present work
*- Lin and Shah (1978)

Table 3.b: Asymptotic Nusselt numbers for the case of prescribed wall heat flux

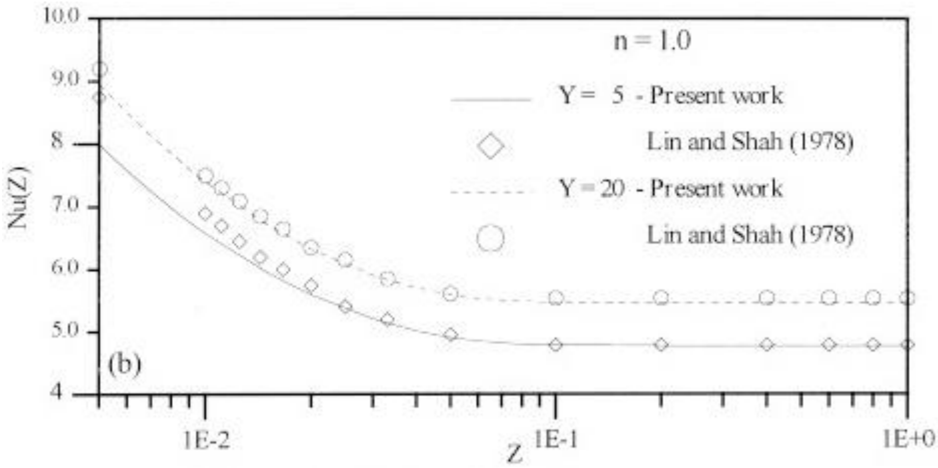
Parallel-Plates Channel								
Y	**Nu_∞**							
	n = 0.5		**n = 0.75**		**n = 1**		**n = 1.5**	
0	8.7568+	8.762*	8.4275+	8.431*	8.2353+	8.239*	8.0201+	8.024*
1	9.0739	9.080	8.6037	8.609	8.3306	8.335	8.0475	8.051
5	9.8097	9.818	9.1225	9.127	8.6611	8.666	8.1592	8.163
10	10.256	10.27	9.5155	9.522	8.9610	8.966	8.2902	8.294
20	10.693	10.71	9.9652	9.974	9.3520	9.357	8.5088	8.513
Circular Tube								
Y	**Nu_∞**							
	n = 0.5		**n = 0.75**		**n = 1**		**n = 1.5**	
0	4.7458+	4.743*	4.5012+	4.498*	4.3636+	4.362*	4.2141+	4.212*
1	5.0337	5.034	4.6601	4.659	4.4510	4.449	4.2411	4.240
5	5.7474	5.789	5.1586	5.182	4.7724	4.783	4.3552	4.356
10	6.1953	6.292	5.5465	5.605	5.0727	5.107	4.4952	4.502
20	6.1953	6.292	5.5465	5.605	5.0727	5.107	4.4952	4.502

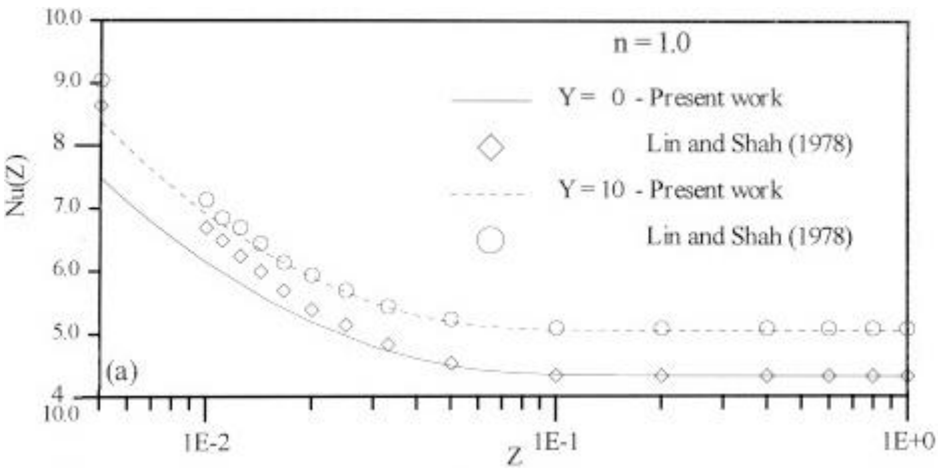
+ - Present work
* - Lin and Shah (1978)

For the case of prescribed wall heat flux in a circular duct, Figure 1 shows the evolution of the local Nusselt number as a function of the axial coordinate, Z, in the thermal entry region. The power-law exponent here considered was n = 1 (newtonian situation) and the yield numbers were Y = 0, 5, 10 and 20. The analysis permits the comparison among the results from the present work and those obtained by Lin and Shah (1978) over the range $5x10^{-3} \leq Z \leq 1$, and those by Nouar et al. (1994) over the range of $5x10^{-6} \leq Z \leq 10^{-1}$. From this figure only a reasonable agreement is observed between the results obtained by the two analyses, with a better agreement for increasing axial coordinate Z, i.e., only in the vicinity of the fully-developed region. Therefore, the two previous works, that have employed finite-differences schemes could not obtain numerical results with high accuracy in the thermal entry region.

In Figures 2 and 3 the local Nusselt numbers are shown in the thermal entry region for the case of prescribed wall temperature for both adopted geometries and for various power-law exponents and yield numbers. The increase of the power-law exponent shows a Nusselt number decrease, while for an increasing yield number there is an increase in the Nusselt number. These results are systematically larger for parallel-plates channels than for circular tubes.

Figures 4 and 5 present a similar analysis for the case of a prescribed wall heat flux and the same observations are noted in relation to the parameters studied, i.e., power-law exponents, yield numbers and geometries of the channels. Finally, from these figures it is also observed that the Nusselt numbers for the case of a prescribed wall heat flux are larger than those for the case of a prescribed wall temperature.





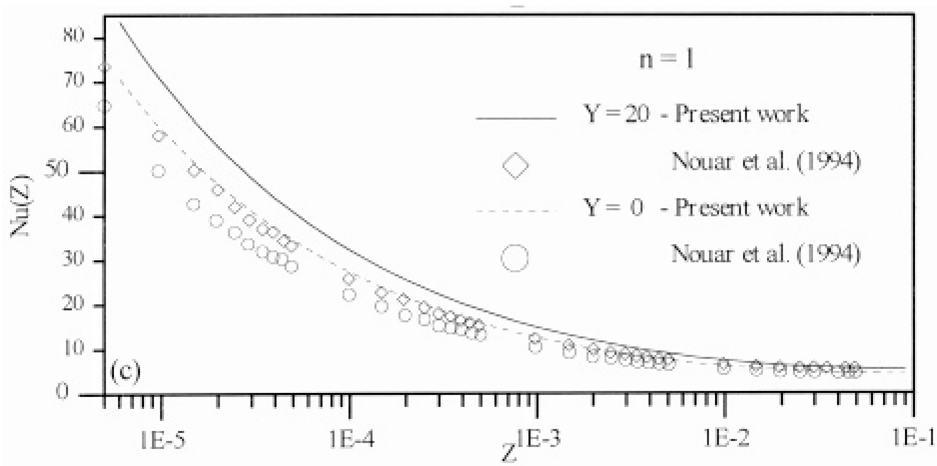
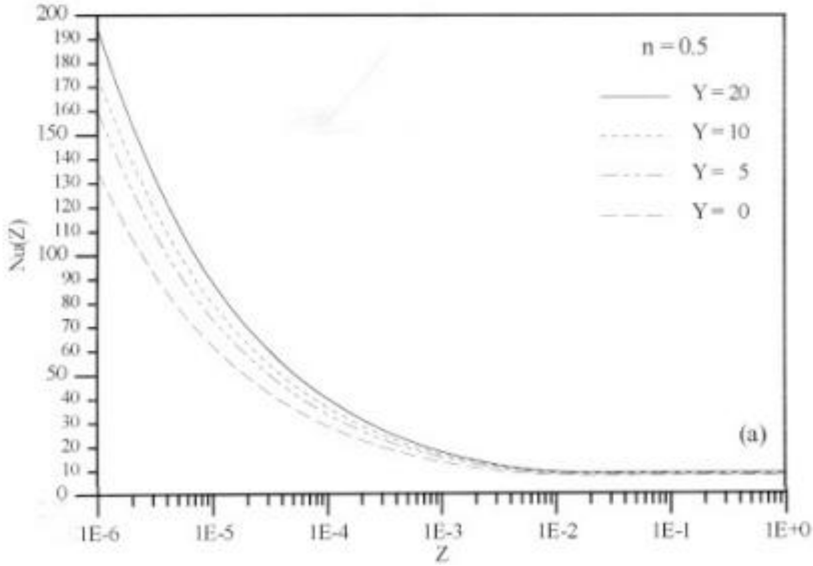


Figure 1: Comparison of the Nusselt number in the thermal entry region for prescribed wall heat flux in a circular tube.



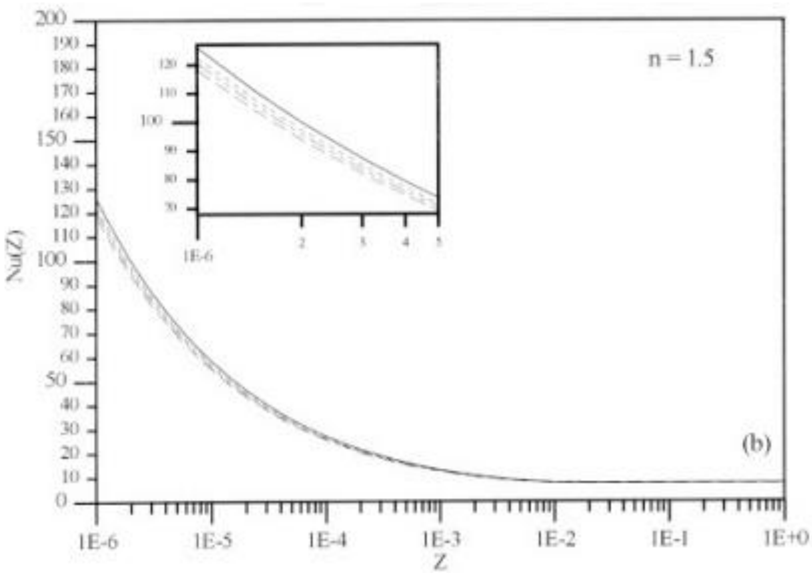


Figure 2 - Local Nusselt number in the thermal entry region for prescribed wall temperature in a parallel-plates channel; (a) n = 0.5; (b) n = 1.5

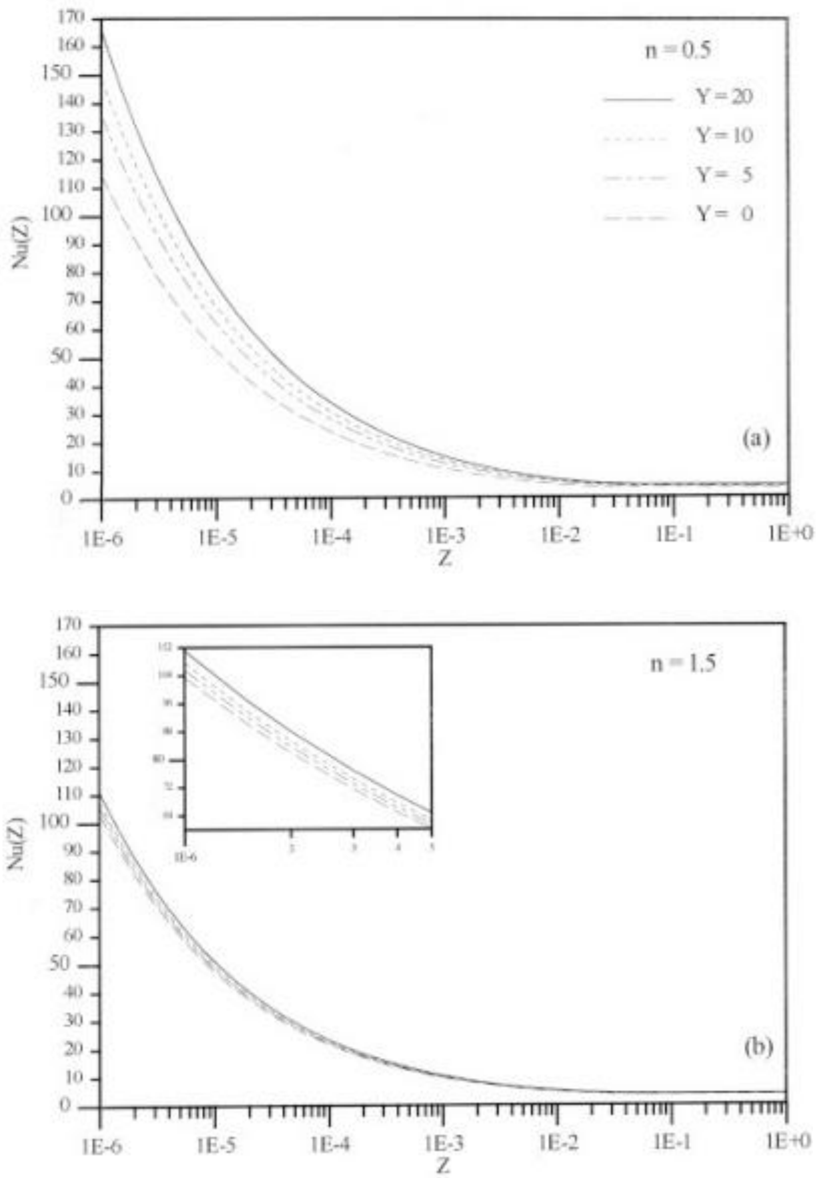


Figure 3 - Local Nusselt number in the thermal entry region for prescribed wall temperature in a circular tube; (a) n = 0.5; (b) n = 1.5

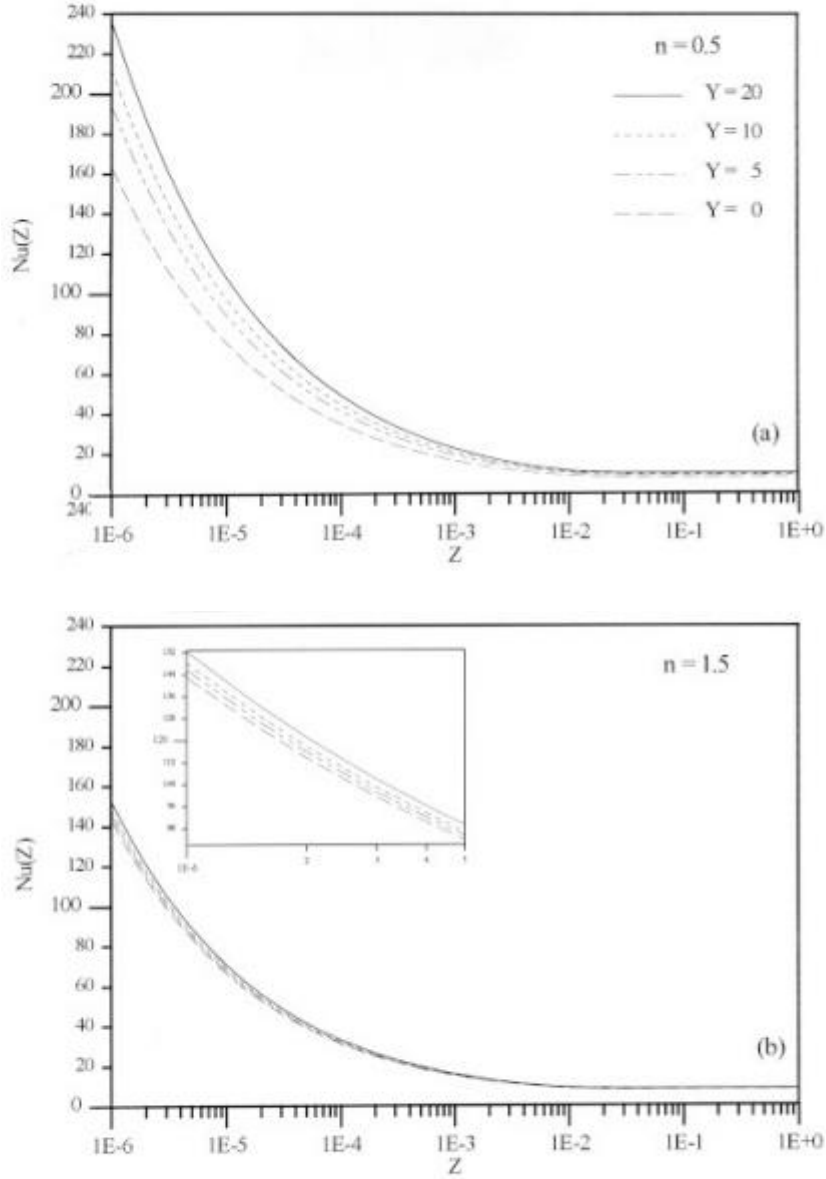


Figure 4 - Local Nusselt number in the thermal entry region for prescribed wall heat flux in a parallel-plates channel; (a) n = 0.5; (b) n = 1.5

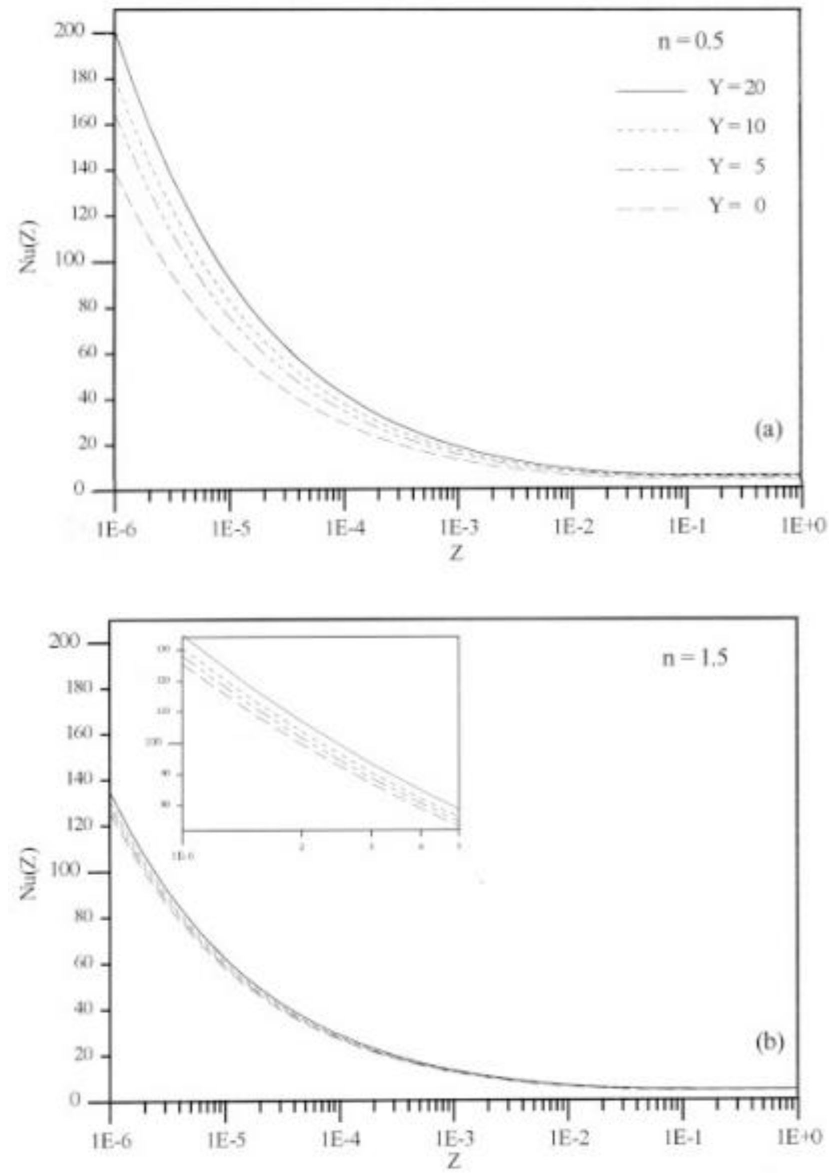


Figure 5 - Local Nusselt number in the thermal entry region for prescribed wall heat flux in a circular tube; (a) n = 0.5; (b) n = 1.5

CONCLUSIONS

The problem of laminar convective heat transfer in the thermal entry and fully-developed flow regions of a Herschel-Bulkley fluid, for both prescribed wall temperature and prescribed wall heat flux, and for circular and parallel-plates channel, has been analyzed, with excellent computational performance, through the Integral Transform Technique in conjunction with the Sign-Count method and

Generalized Integral Transform Technique (GITT) approaches for the solution of the related eigenvalue problem.

Benchmark results are then tabulated and graphically presented for various power-law exponents and yield numbers.

NOMENCLATURE

b Radius of circular duct or one half the spacing between parallel-plates, m
b_o Radius of plug-flow region, m
D_h Hydraulic diameter, m
f Fanning friction factor
h (z) Local heat transfer coefficient, $W/m^2.K$
k Thermal conductivity, W/m.K
K Consistency index of the fluid, $N.s^n/m^2$
m Coefficient defined in equations (7)
n Power-law exponent
N_i Normalization integral
Nu (Z) Local Nusselt number
Nu_∞ Asymptotic Nusselt number
p Parameter of channel geometry or pressure
Pr Prandtl number
q_w Prescribed wall heat flux, W/m^2
r, R Radial coordinate, dimensional and dimensionless
Re Reynolds number
R_o Dimensionless radius of plug-flow region
T_o Inlet temperature (K)
T_w Prescribed wall temperature, K
u_m Average flow velocity, m/s
u(r), U(R) Velocity distribution, dimensional and dimensionless
W (R) Defined by equations (8)
Y Yield number
z, Z Axial coordinate, dimensional and dimensionless

Greek Symbols

α Fluid thermal diffusivity, m^2/s
$\dot{\gamma}$ Shear rate, s^{-1}
μ_i Eigenvalues of problem (18)
$\psi_i(R)$ Eigenfunctions of problem (18)
ρ Density, Kg/m^3
τ Shear stress, N/m^2
τ_y Yield stress, N/m^2

$\theta(R, Z)$ Dimensionless temperature distribution

$\theta_{av}(Z)$ Dimensionless average temperature

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