A Law of the Wall Formulation for Recirculating Flows

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This work presents a new law of the wall formulation for recirculating turbulent flows. An alternative expression for the internal length which can be applied in the separated region is also presented. The formulation is implemented in a numerical code which solves the k-ε model through a finite volume method. The theoretical results are compared with the experimental data of Vogel and Eaton (J. of Heat Transfer, Transactions of ASME, vol.107, pp. 922-929, 1985). The paper shows that the present formulation furnishes better results than the standard k-ε formulation.
Keywords: *Turbulence, recirculating flows, law of the wall.*

Introduction

In past years, the k-ε model has become one of the most popular turbulence models, having been applied to a host of practical problems. This popularity has arisen mainly due to the combination of two factors: i) a relatively simple implementation procedure and, ii) a certain degree of generality in application. Despite these advantages, the k-ε model still presents some difficulties, in particular, in the description of near wall flows.

To overcome these difficulties, several schemes were proposed which can be basically divided into two groups. In the first group (see, e.g., Chieng and Launder(1980), Ciofalo and Collins(1989)), the so-called high Reynolds number models, analytic solutions are advanced for the near wall region which can the used as boundary conditions in numerical simulations of flow; they require a coarse grid. In the second group (see Patel, Rodi and Sheuerer(1985)), the low Reynolds number models, dumping functions are used to model the turbulence right down to the wall where the viscous effects dominate; they require a fine grid and, consequently, large computational time. The two groups present distinct features. Models that lie in the first group normally converge easily and are robust; they do not, however, describe well flows in a recirculation region. Models in the second group, as mentioned before, are computationally expense and difficult to converge. Much of the difficulty with the high Reynolds number models lies in the manner in which the near wall analytical solution is prescribed. The classical logarithmic expression commonly used is an exact result for flow over a flat plate, so that it does not apply for flow in separating and recirculating flow regions.

In the present work, a new version of the classical law of the wall will be presented which is general enough to be employed all along the flow, including any existing separating and recirculating flow regions. An alternative formulation for the characteristic length in the near wall region is also presented (Cruz and Silva Freire, 1995). All results here presented are implemented in a numerical code to describe the flow behind a back-ward facing step (Figure 1). A comparison with the experimental data of Vogel and Eaton(1985) and with the standard k-ε model of Spalding and Launder(1974) is then presented.

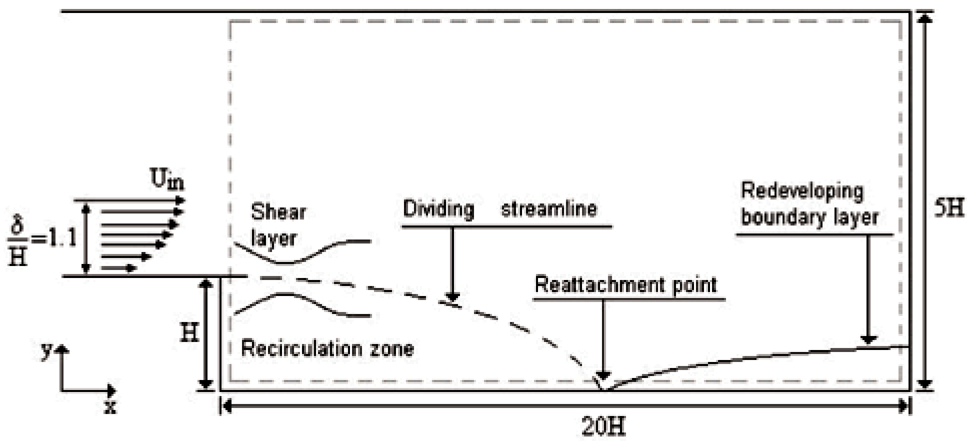

Figure 1. Back step geometry and computacional domain.

Governing equations

In the present analysis, the averaged Navier-Stokes equations for an incompressible flow were used. These were complemented by the turbulent kinetic energy equation and the mean dissipation rate equation which are show below.

$$\frac{Dk}{Dt}=\frac{\partial}{\partial x_i}\left[\frac{\nu_T}{\sigma_K}\frac{\partial k}{\partial x_i}\right]+\nu_T\left[\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right]\frac{\partial U}{\partial x_j}-\varepsilon \tag{1}$$

$$\frac{D\varepsilon}{Dt}=\frac{\partial}{\partial x_i}\left[\frac{\nu_T}{\sigma_t}\frac{\partial \varepsilon}{\partial x_i}\right]+C_1\frac{\varepsilon}{k}\nu_T\left[\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right]\frac{\partial U}{\partial x_j}-C_2\frac{\varepsilon^2}{k} \tag{2}$$

where $\nu_T=C_\mu\frac{k^2}{\varepsilon}$; $C_\mu=0{,}09$, $C_1=1{,}44$;$C_2=1{,}92$; $\sigma_t=1{,}3$; $\sigma_K=1$.

Law of the wall formulation

In the near wall region, where the inertia terms are small, the momentum equation can be approximated by the following relation:

$$\nu\frac{\partial u}{\partial y}+\overline{u'v'}=\frac{\tau_w}{\rho}+\frac{1}{\rho}\frac{dP}{dx}y \tag{3}$$

where ν is the kinematic viscosity, τ_w is the shear stress at the wall, and is P the pressure. $\overline{u'v'}$ represents the turbulent shear stress and ρ is the density.

In the above equation, the left hand side represents the total shear stress, which is furnished by the sum of the laminar and turbulent terms as described below:

$$\nu\frac{\partial u}{\partial y}+\overline{u'v'}=\tau\ . \tag{4}$$

To evaluate the wall shear stress a simplified version of the law of the wall proposed by Cruz and Silva Freire (1995) will be used. In addition, an alternative equation for the characteristic length in the inner regions of the flow will be used. Both equations are shown below.

$$u=\frac{\tau_w}{|\tau_w|}\frac{2}{k}\sqrt{\frac{\tau_w}{\rho}+\frac{1}{\rho}\frac{dP}{dx}y}+\frac{\tau_w}{|\tau_w|}\frac{u_\tau}{k}\ln\left(\frac{y}{L_C}\right) \tag{5}$$

$$L_C = \frac{-\frac{\tau_w}{\rho} + \sqrt{\left(\frac{\tau_w}{\rho}\right)^2 + 2\nu \frac{dP}{dx} u_R}}{\frac{1}{\rho}\frac{dP}{dx}} \tag{6}$$

where k Vón Karmán's constant (=0.41), $u_\tau = (\tau_w / \rho)^{1/2}$ is the friction velocity and u_R is given by the following equation:

$$u_R = \sqrt{\tau} \tag{7}$$

Equations (5) and (6) furnish a generalised law of the wall that can be applied to any flow region. Far away from a separation point, the wall shear stress is positive and dP/dx y <<τ_w. Thus, equations (5) and (6) are reduced to:

$$u = \frac{2}{k} u_\tau + u_\tau \ln\left(\frac{y}{L_c}\right) \tag{8}$$

$$L_C = \nu / u_\tau \ . \tag{9}$$

That is, the classical law of the wall. Near to a separation point, where τ_w=0 the equation reduces to:

$$u = \frac{2}{k}\sqrt{y\frac{dP}{\rho dx}} \ . \tag{10}$$

Equation (9) is similar to the expression proposed by Stratford (1959), which describes the velocity profile in the near wall region of a turbulent flow that approaches a separation point, that is, in the flow region where the wall shear stress tends to zero.

In the flow recirculation region where dP/dx y >>τ equations (5) and (6) can be written as

$$u = -\frac{2}{k} u_\tau - \frac{u_\tau}{k} \ln\left(\frac{y}{L_C}\right) \tag{11}$$

$$L_C = 2\left|\tau_w \Big/ \frac{dP}{dx}\right| . \tag{12}$$

The main difference between equations (6) and (7) and equations (10) and (11), besides the negative sign related to the flow direction, is the flow characteristic length near to the wall (equation (11)), which differs from the classical characteristic length given by equation (8). A major difference is that now the characteristic length in the reverse flow region depends directly on the pressure gradient. The experimental works of Simpson et al.(1981) and Thompson and Whitelaw (1985) showed that the classical characteristic length in the wall region, ν/u_τ , is not appropriated to describe the separation phenomenon in the reverse flow region. Besides, according to Simpson et al.(1981), whatever the value of $L_C$in the reverse flow region, the characteristic length cannot vary inversely with the shear stress at the wall, as does the classical characteristic length. Here, we point out that as the pressure gradient tends to zero, equation (6) reduces to the classical characteristic length equation.

With a view to avoid an additional iterative numerical procedure, which could make it difficult for the computer code to converge, the following set of equations was used for the evaluation of the shear stress at the wall.

$$\tau = C_\mu^{1/2} k + \nu \frac{\partial u}{\partial y} \tag{13}$$

$$\tau_{wo} = \frac{u\sqrt{\tau}\,\rho \bar{k}}{\ln\left(Ey\dfrac{\sqrt{\tau}}{\nu}\right)} C_\mu^{1/4} \tag{14}$$

$$\frac{1}{\rho}\frac{dP}{dx} = \frac{\tau - \tau_{wo}}{y} \tag{15}$$

$$L_C = \frac{-\dfrac{\tau_{wo}}{\rho} + \sqrt{\left(\dfrac{\tau_{wo}}{\rho}\right)^2 + 2\nu\dfrac{dP}{dx}u_R}}{\dfrac{1}{\rho}\dfrac{dP}{dx}} \tag{16}$$

$$\tau_w = \frac{u\sqrt{\tau}\,\rho \bar{k}}{\left[2\sqrt{\left|\dfrac{\tau}{\tau_{wo}}\right|} + \ln\left(\dfrac{y}{L_C}\right)\right]} \tag{17}$$

where E=9.8

The above set of equations results in a linearized procedure for the evaluation of the shear stress at the wall, what makes the calculations more robust and easier convergence of the numerical code. A first estimate for the wall shear stress is obtained through equation (14), which is next used to linearize the calculation of the wall shear stress furnished by equation (17). The calculation of the pressure gradient is also made indirectly through equation (15).

The above formulation was employed for $y^+ > 5$ where $y^+ = y u_\tau / \nu$; below this value a completely viscous region was considered to exist. Usually, the value $y^+ > 11$ is adopted to delimit the viscous region; this value is then normally taken as a lower bound for an application of the logarithmic law of the wall (Chieng and Launder(1980) Ciofalo and Collins(1989)). This value, however, was obtained for flow over a flat plate subjected to a zero pressure gradient; it is thus not valid in the regions of separated and reverse flows. A detailed analysis on the appropriate value that must be taken to correctly represent the boundary between the law of the wall region and the viscous region in the separation and reverse flow regions, can only be made with the help of experimental data which are still scarce in literature. The present study suggests that the search for this value must be made in terms of the values of y/L_c and not of y^+.

The dissipation and the production terms for the turbulent kinetic energy are then given by

$$\varepsilon = C_\mu^{1/2} k \left[\frac{\sqrt{\tau}}{\bar{k} y} + \frac{\frac{1}{\rho}\frac{dP}{dx}}{\bar{k}\sqrt{\tau}} \right] \tag{18}$$

$$PRODUCTION = \frac{C_\mu^{1/2} k \rho}{y} \left[\frac{2\sqrt{\tau}}{\bar{k}} + \frac{\sqrt{|\tau_{wo}|}}{\bar{k}} \ln\left(\frac{y}{L_C} \right) \right]. \tag{19}$$

Results

The present formulation was used in the calculation of a back-ward facing step flow. The flow conditions are the same as those presented in the work of Vogel and Eaton(1985). The governing equations are discretized using a finite volume formulation coupled with a hybrid scheme for the treatment of the convective and diffusive terms simultaneously. The resulting set of equations was solved iteratively through a robust and extensively tested version of the numerical code TEACH-2E (Teaching Elliptic Axi-Symmetrical Characteristic Heuristically), which incorporates the algorithm SIMPLE, specific for pressure-velocity coupling in incompressible flows. The best relation for CPU time versus accuracy was obtained for a grid with 146x102 points.

In the next two figures, the velocity profiles in the near separation point region are shown. A comparison between the present formulation and results obtained with the standard k-εmodel of

Launder and Spalding (1974) and the experimental data of Vogel e Eaton (1985) is made. The present formulation is clearly superior to the standard formulation, furnishing better results for the very near wall region. In fact, the calculations performed with the standard model needed a larger numbr of iterations to converge (1200) that the present model (1100). We point out that the proposed procedure pictures well the small secondary recirculation region present in the flow near to the vertical face of the step (Figure 1). Other formulations that employ the classical logarithmic law of the wall in the original k-εmodel equations (Launder and Spalding, 1974) but divide the flow near the wall in several layers give better results that the classical formulation but are still unable to reproduce the secondary recirculation region (Bortolus and Giovannini, 1995).

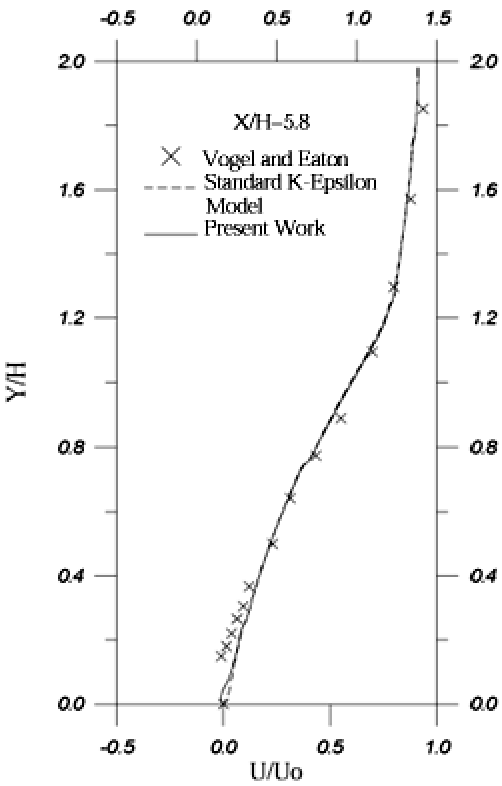

Figure 2. Comparison of the Velocity Profile for X/H=5.8.

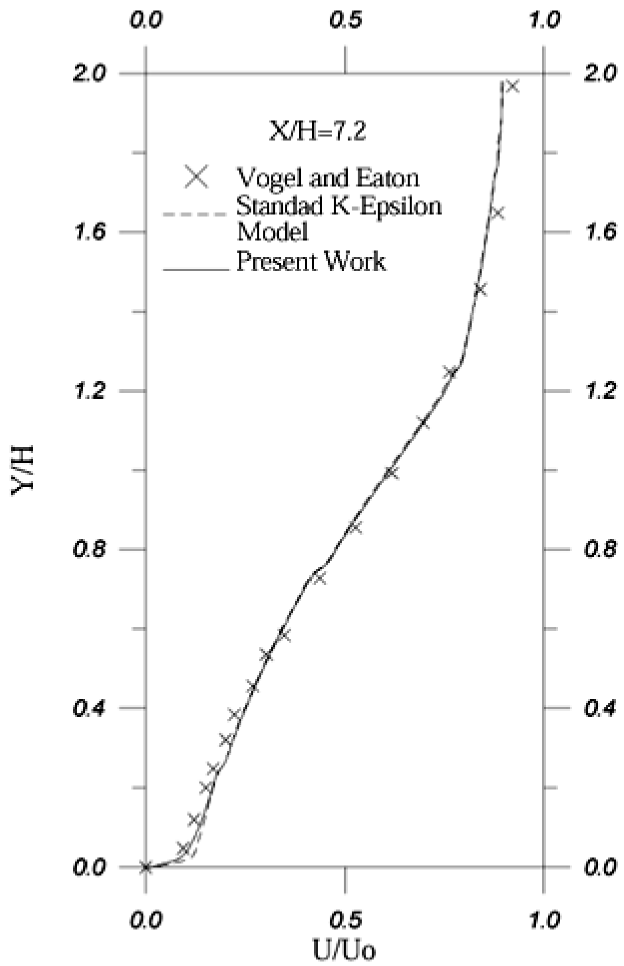

Figure 3. Comparison of the Velocity Profile for X/H=7.2.

The figure 4 compares the friction coefficient obtained with the procedure here developed with results obtained with the standard k-ε model and the experimental data. Again, the present results give a better representation of the phenomenon before and after the reattachment point.

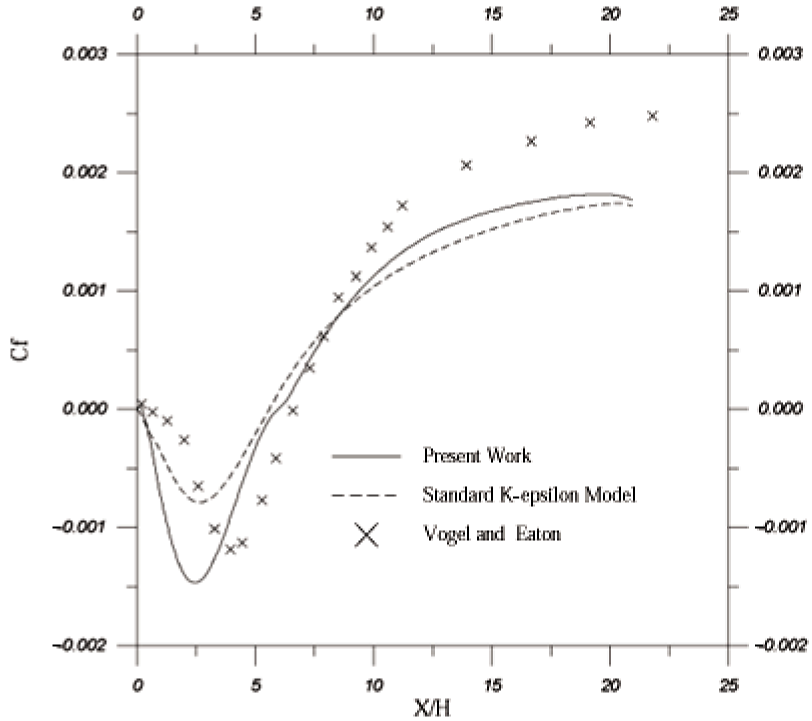

Figure 4. Skin-friction coefficient comparison for several cases.

In Table 1, values of the reattachment point obtained by both theories, together with the experimental data are shown. The calculation made with the proposed theory furnishes a length for the recirculation zone closer to the experimental value when compared with the value obtained through the k-ε standard model. This happens because equation (17) gives higher values for the dissipation rate close to the wall causing a decrease in the turbulent viscosity in this region, what causes an increase in the recirculation region

Table 1. Reattachment point position.

Reattachment Position by	X/H
Experiments, Vogel and Eaton(1985)	6.6
Present work	6.0
Standard K-ε model	5.5

In figures 5 and 6 the near wall behavior of the root mean square of the velocity calculated by the present formulation, is compared with the experimental data and the standard k-epsilon formulation for two stations in the redevelopment region. The present work reproduces much better the experimental data, giving higher levels of the velocity fluctuation than the standard k-epsilon model, in the very near wall region. This better agreement occurs basically, because the proposed formulation takes into account the effects of the pressure gradient in the dissipation and production terms (equations 18 and 19), in the region after the reattachment of the flow.

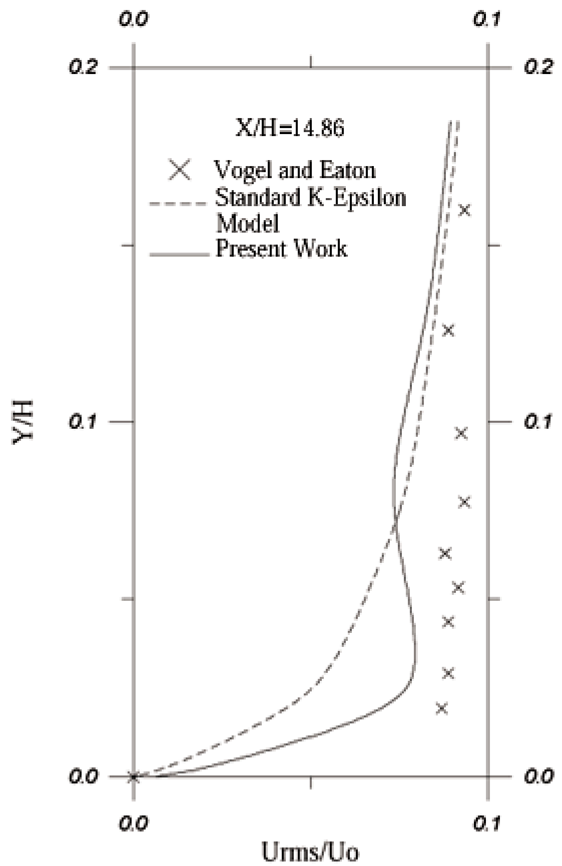

Figure 5. Near wall behavior of the root mean square of the velocity at X/H =14.86.

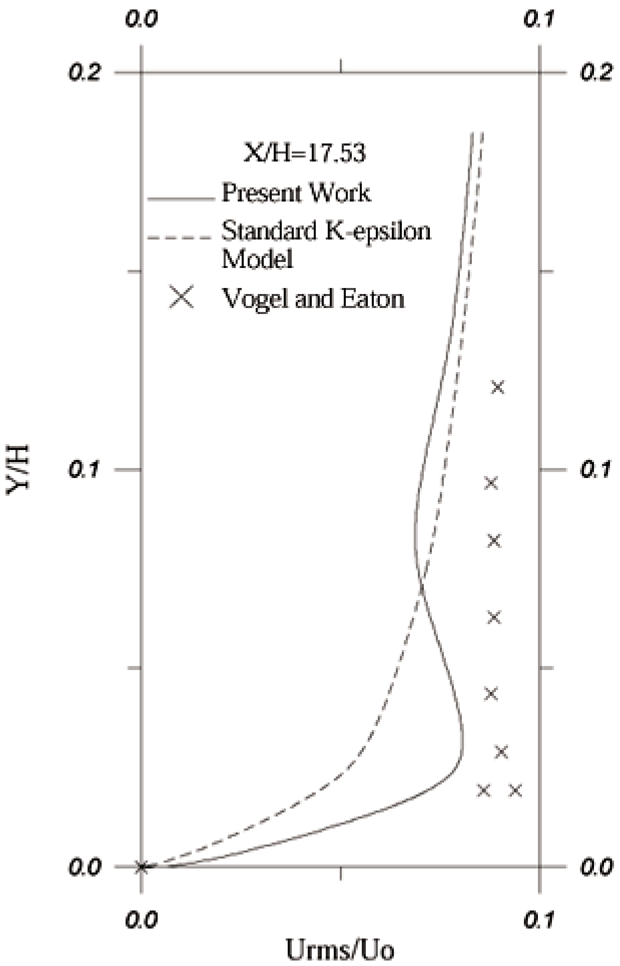

Figure 6. Near wall behavior of the root mean square of the velocity at X/H =17.53.

Conclusion

In this work, a new formulation was presented for the description of the flow near to a reattachment point. This formulation was implemented in a computer code which solves the averaged Navier-Stokes equations together with the k-ε model through a finite volume method. The present formulation was shown to furnish better results than the classical formulation at a smaller computational cost.

Thus, the results show that complex turbulent flows subjected to separation, reattachment and recirculating regions can be adequately described through the use of appropriate law of the wall formulation. In addition, the use of the law of the wall formulation results in codes which are robust and cheap to run, an important feature in the solution of engineering problems.

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