

Slabs strengthened for punching shear with post-installed steel and CFRP connectors

Lajes reforçadas à punção com conectores de aço e PRFC pós-instalados



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Abstract

Structural accidents due to punching shear failures have been reported in flat slab buildings. Design recommendations presented by codes can lead to entirely different punching shear resistance estimates for similar situations. Furthermore, design codes do not present guidelines for the design of punching shear strengthening of existing slabs. This paper uses a database with 118 experimental results to discuss the performance of theoretical estimates of punching shear resistance using ACI 318, Eurocode 2 and ABNT NBR 6118 in the case of slabs without shear reinforcement. Another database with results of 62 tests on slabs strengthened with post-installed steel and CFRP dowels is used to evaluate the performance of these strengthening techniques and to propose adaptations in codes to allow their use in punching shear strengthening situations of existing slab-column connections.

Keywords: flat slabs, punching shear, structural strengthening; CFRP, post-installed steel connectors.

Resumo

Acidentes estruturais por punção vêm sendo relatados em edifícios com lajes lisas. As recomendações de projeto apresentadas pelas normas podem levar a estimativas de resistência à punção divergentes para situações semelhantes. Além disso, não são apresentadas orientações para o dimensionamento do reforço à punção de lajes existentes. Este artigo utiliza um banco de dados com 118 resultados experimentais para discutir o desempenho das estimativas teóricas de resistência à punção obtidas usando o ACI 318, o Eurocode 2 e a ABNT NBR 6118 para o caso de lajes sem armadura de cisalhamento. Um outro banco de dados, com resultados de 62 ensaios em lajes reforçadas com conectores pós-instalados de aço e PRFC, é utilizado para avaliar o desempenho destas técnicas de reforço e para apresentar propostas de adaptação das recomendações destas normas para permitir seu uso em situações de reforço à punção de ligações laje-pilar existentes.

Palavras-chave: lajes lisas, punção, reforço estrutural, PRFC, conectores pós-instalados de aço.

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1. Introduction

Failures in design, construction, use and maintenance phases or changes in the purpose of a building are some of the reasons that can lead to structural strengthening. In flat-slab buildings, the slab-column connection is a critical point due to punching shear, which is a brittle failure mode that can bring the structure to fail through progressive collapse. This structural system was developed in the early 20th century and simplifies formwork and reinforcement production, but requires attention since several cases of accidents have been reported. Melo and Regan [1] report that the first structural accident caused by punching shear was of Prest-o-Lite building, which occurred in Indianapolis in 1911. Since then, other cases have been reported in the literature.

Figure 1a presents the case of the collapse of 2000 Commonwealth Avenue building. It was a 16-storey apartment building that collapsed during its construction in 1971 in the city of Boston, USA, victimising

four workers. King and Delatte [2] present a review of the case and conclude that the accident was caused by the local failure of one slab-column connection of the roof slab, which spread to a large area of the building. During the investigation process, several mistakes and omissions were observed regarding design and construction. In Figure 1b it is possible to see the case of Bullock's Department Store building, whose structure was composed of waffle slabs supported on circular columns. According to Mitchel *et al.* [3], the collapse occurred in 1994 after an earthquake in California and the lack of post-punching reinforcement caused failure get to spread. Gardner [4] show the causes of the collapse of the Sampoong Department Store (see Figure 1c), which occurred in 1995 in South Korea and concludes that the accident was caused by design and execution failures, leading to 502 fatalities and 937 injuries. Another example of punching shear collapse happened in the Piper Rows Car Park building, shown in Figure 1d, which occurred in 1997 in England, mainly due to corrosion of the flexural reinforcement, as reported by Woods [5].



a) 2000 Commonwealth Avenue, 1971
 Author: King and Delatte [2]



b) Bullock's Department Store, 1994
 Author: www.johnmartin.com [3]



c) Sampoong Department Store, 1995
 Author: Gardner *et al.* [4]



d) Piper Rows Car Park, 1997
 Author: Woods [5]

Figure 1
 Structural accidents caused by punching shear



Structure overview post-collapse

Connections details before and post-collapse

a) Shopping Mall Rio Poty, 2013. **Author:** Oliveira *et al.* [6]



Structure overview post collapse

Position detail of reinforcements and tendons on slab-column connection

b) Residencial Grand Parc, 2016. **Author:** Coutinho *et al.* [7]

Figure 2 Structural accidents caused by punching shear in Brazil

In Brazil, two recent accidents, caused by punching shear have been recorded. In the city of Teresina, Piauí, an area of 40,000 m² of Rio Poty Shopping Centre (see Figure 2a) failed during its construction in 2013, with no fatalities. In 2016, in Vitoria, Espírito Santo, the collapse of the leisure area of the residential building Grand Parc (see Figure 2b) occurred, leading to one fatal victim. In both cases, the technical documents available to date (see Oliveira *et al.* [6] and Coutinho *et al.* [7]) are not conclusive but point to several failures in the construction phase of these structures.

The literature review indicates that many of the structural accidents occurring in buildings with flat slabs begin in a localised way, by punching shear, originating from design and construction failures. Soares and Vollum [8] broadly discuss the differences between the current recommendations and those previously used in the United Kingdom for punching shear design of concrete flat slabs and point out that design codes can lead to significantly different resistance estimates for similar situations. This may favour divergences

during the design or assessment of a building's resistance. Koppitz *et al.* [9] warn that in cases where there is a need to increase the strength of the structure, the situation is even more critical since there are no code recommendations to guide the professionals involved about the strengthening techniques and calculation methods that must be used.

This paper discusses the performance of international and the Brazilian codes in the assessment of the punching shear resistance of slab-column connections without shear reinforcement. This is done using ACI 318 [10], ABNT NBR 6118 [11] and Eurocode 2 [12, 13, and 14], through comparison between theoretical predictions and experimental resistances. The performance of the codes is evaluated using a broad and updated database, containing carefully selected results from research conducted in Brazil and abroad. The objective is to show the context in which the recommendations currently employed in Brazil are found, providing to the technical community parameters to establish criteria, in the

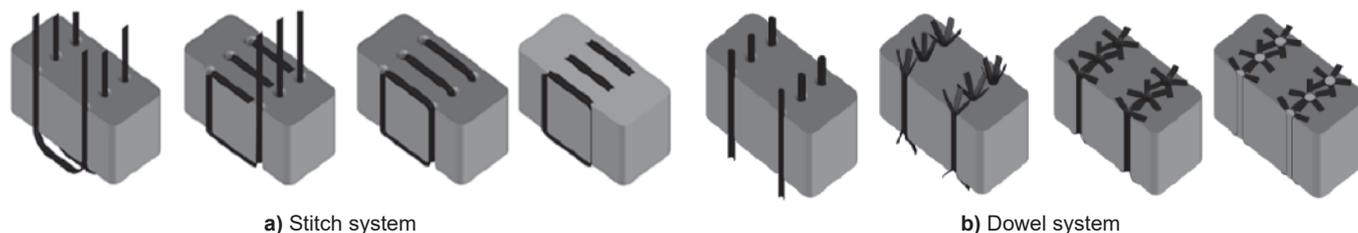


Figure 3
Punching shear strengthening of slab-column connections with CFRP (adapted from Santos [16])

absence of specific national standardisation for assessment of the punching capacity of flat-slab buildings. After these analyses, a new database, bringing together experimental results of tests on slabs strengthened for punching shear with post-installed steel and CFRP connectors, is presented. These results are used to propose adaptations in the design codes so they can be used to design the punching strengthening of reinforced concrete slab-column connections with steel and PRFC post-installed connectors.

2. Theoretical basis

2.1 Punching shear strengthening techniques

Carbon fibre reinforced polymers (CFRP) can be used in different ways for the punching strengthening of slab-column connections. According to Sissakis and Sheikh [15], they can contribute to increasing both the resistance as well as maximum strain capacity in case collapse. According to Santos [16], the flexible nature of this material allows it to be fixed in different forms, being able to be anchored in a loop shape, in an international technique known as stitch, or being used in a way similar to shear connectors, in a method called dowel, with the anchoring made on the slab surfaces, as shown in Figure 3.

In the stitch technique, the CFRP sheets are cut into strips, saturated with resin and inserted into the slab through holes, forming closed loops similar to stirrups (see Figure 3a). After their placement, the holes must be filled with epoxy resin or high-performance mortar to favour the transference of forces between concrete and the surface of the CFRP. The dowel technique, according to Erdogan *et al.* [17], consists of producing dowels from

the cut of CFRP sheets in rectangular sheets, as shown in Figure 3b. After saturation with epoxy resin, the CFRP sheets are rolled, forming a kind of tube. These tubes are installed inside holes in the slab with the aid of a guide, removed soon after the positioning of the strengthening. Subsequently, the upper and lower ends of the CFRP tube are cut and opened in petal-shaped form and bonded to the surface of the slab to ensure anchorage by filling the holes with epoxy resin or high-performance mortar.

Another option for the punching shear strengthening of existing slab-column connections involves the use of post-installed steel connectors. Different types of connectors are industrially commercialised, and Figure 4a illustrates a model where anchoring is done through a nut and washer system. This strengthening technique can increase both resistance and ductility of slab-column connections. It can also be used with a combination of mechanical anchoring on the bottom surface and epoxy adhesive as a bond mechanism, with the dowels vertically installed (see Figure 4b) or inclined (see Figure 4c), as presented by Ruiz *et al.* [18].

2.2 Methods to estimate the punching shear resistance

ACI 318 [10], ABNT NBR 6118 [11] and Eurocode 2 [12, 13 and 14] present recommendations for the design of reinforced and prestressed concrete flat slabs. In general, these standards assume that the punching shear resistance of slabs without shear reinforcement ($V_{R,c}$) can be estimated based on a stress strength (τ_R) acting in a control area ($u_1 \cdot d$). In the case of slabs with shear reinforcement, these codes recommend that the resistance shall be checked for failures occurring: within the shear reinforced zone

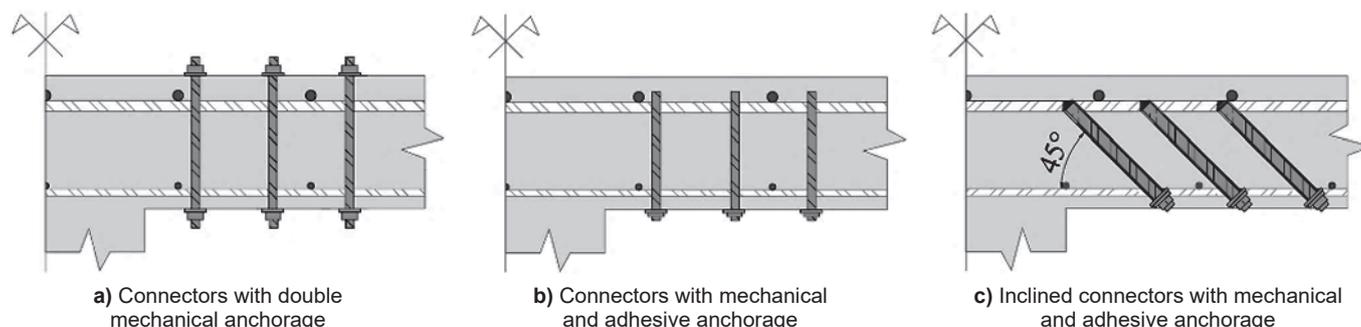


Figure 4
Punching shear strengthening of slab-column connections with post-installed steel connectors



Figure 5
Punching shear failure modes in concrete flat slabs with shear reinforcement

($V_{R,cs}$); in the outside of the area containing shear reinforcement ($V_{R,out}$); in the vicinity of the column due to crushing of the concrete strut ($V_{R,max}$). Figure 5 presents images of these failure modes as described in the literature.

There are no code recommendations to estimate the punching shear resistance of slab-column connections strengthened with post-installed steel or CFRP connectors. In the case of post-installed steel connectors, it is usual to assume that, if installation mechanisms are efficient, the same criteria established for pre-installed reinforcement are valid. In the case of strengthening with CFRP, ACI 440.2R [19] is the primary reference and presents recommendations for shear reinforcement applications in beams and columns, but not for flat slabs.

In cases where shear strengthening involves the structural element entirely, ACI 440.2R [19] recommends that the maximum deformation in the fibre shall be limited to 0.004 for the design. This limitation is based on the practical observation that, in the case

of shear, before the fibre failure, the concrete contribution came from aggregate interlock is lost, as reported by Priestley *et al.* [20]. Table 1 presents a summary of the normative recommendations for the prediction of punching shear resistance of slabs without shear reinforcement. It also presents adaptations proposed for these codes so they can be used for design in strengthening situations. The safety factors used to reduce the resistance of the CFRP in the adjustments of Eurocode 2 [12, 13 and 14] and ABNT NBR 6118 [11] are based on the values proposed by *fib* Bulletin 14 [21]. Figure 6 illustrates the control perimeters used in the calculation of the punching resistance of the slabs in the databases.

3. Evaluation of the performance of theoretical methods

The safety factors were removed from all equations summarised in Table 1 to evaluate the performance of the theoretical punching

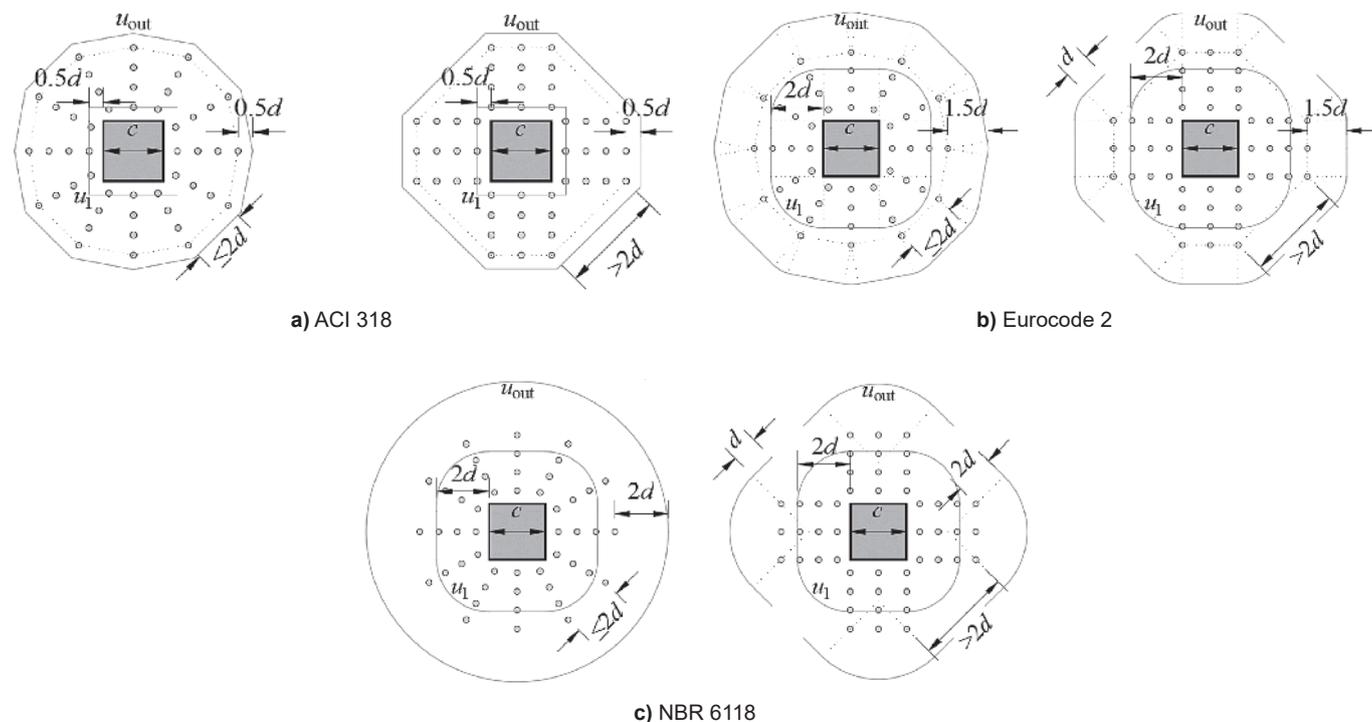


Figure 6
Control perimeters

Table 1

Summary of the methodology to estimate the punching shear resistance of slabs

Code	Slabs without shear reinforcement	Slabs strengthened for shear
ACI 318	$V_{Rdc} = \phi \cdot \tau_{Rc} \cdot u_1 \cdot d$ $\tau_{Rc} = \min \begin{cases} 0.083 \cdot (40 \cdot d/u_1 + 2) \cdot \sqrt{f_c'} \\ 0.17 \cdot (1 + 2/[a/b]) \cdot \sqrt{f_c'} \\ 0.33 \cdot \sqrt{f_c'} \end{cases}$ $\phi = 0.75$	$V_{Rd} = \min \begin{cases} V_{Rdcs} = \eta_c \cdot V_{Rdc} + \phi \cdot A_{sw} \cdot \sigma_{wd} \cdot (d/s_r) \\ V_{Rdout} = \phi \cdot 0.17 \cdot \sqrt{f_c'} \cdot u_{out} \cdot d \\ V_{Rdmax} = k_{sys} \cdot V_{Rdc} \end{cases}$ $\sigma_{wd} = \begin{cases} f_{yw} \leq 420 \text{ MPa} & \text{(steel connectors)} \\ 0.004 \cdot E_{CFRP} \leq 0.75 \cdot \varepsilon_{fu} \cdot E_{CFRP} & \text{(CFRP)} \end{cases}$ $\eta_c = \begin{cases} 0.50 & \text{(CFRP)} \\ 0.75 & \text{(steel connectors)} \end{cases}$ $k_{sys} = \begin{cases} 1.5 & \text{(CFRRP)} \\ 2.0 & \text{(steel connectors)} \end{cases}$
Eurocode 2	$V_{Rdc} = \tau_{Rdc} \cdot u_1 \cdot d \geq v_{min} \cdot u_1 \cdot d$ $\tau_{Rdc} = \frac{0.18}{\gamma_c} \cdot k \cdot (100 \cdot \rho \cdot f_{ck})^{1/3}$ <p>Where:</p> $v_{min} = 0.035 \cdot k^{2/3} \cdot \sqrt{f_{ck}}$ $k = 1 + \sqrt{200/d} \leq 2$ $\rho \leq 0.02$ $\gamma_c = 1.5$	$V_{Rd} = \min \begin{cases} V_{Rdcs} = 0.75 \cdot V_{Rdc} + A_{sw} \cdot \sigma_{wd} \cdot (1.5 \cdot d/s_r) \leq 1.5 \cdot V_{Rdc} \\ V_{Rdout} = \frac{0.18}{\gamma_c} \cdot k \cdot (100 \cdot \rho \cdot f_{ck})^{1/3} \cdot u_{out} \cdot d \\ V_{Rdmax} = \frac{0.24}{\gamma_c} \cdot f_{ck} \cdot (1 - f_{ck}/250) \cdot u_0 \cdot d \end{cases}$ $\sigma_{wd} = \begin{cases} (250 + 0.25 \cdot d) \leq \frac{f_{yw}}{\gamma_s} & \text{(steel connectors)} \\ \frac{0.004 \cdot E_{CFRP}}{\gamma_{CFRP}} \leq \frac{0.75 \cdot \varepsilon_{fu} \cdot E_{CFRP}}{\gamma_{CFRP}} & \text{(CFRP)} \end{cases}$ $\gamma_s = 1.30 \left(\begin{array}{l} \text{value suggested by the authors in the absence of} \\ \text{experimental validation for each type of strengthening} \end{array} \right)$ $\gamma_{CFRP} = \begin{cases} 1.20 & \text{for applications with high degree of quality control} \\ 1.35 & \text{for applications with normal quality control or} \\ & \text{under difficult on-site working conditions} \end{cases}$
NBR 6118	$V_{Rdc} = \tau_{Rdc} \cdot u_1 \cdot d$ $\tau_{Rdc} = \frac{0.182}{\gamma_c} \cdot k \cdot (100 \cdot \rho \cdot f_{ck})^{1/3}$ <p>where:</p> $k = 1 + \sqrt{200/d}$ $\gamma_c = 1.4$	<ul style="list-style-type: none"> • steel connector $\sigma_{wd} = \begin{cases} f_{ywd} \leq \frac{345}{\gamma_s} \text{ MPa for } h \leq 150 \text{ mm} \\ f_{ywd} \leq \frac{228.75 + 0.775 \cdot h}{\gamma_s} \leq \frac{500}{\gamma_s} \text{ MPa for } h > 150 \text{ mm} \end{cases}$ • CFRP connector $\sigma_{wd} = \frac{0.004 \cdot E_{CFRP}}{\gamma_{CFRP}} \leq \frac{0.75 \cdot \varepsilon_{fu} \cdot E_{CFRP}}{\gamma_{CFRP}}$ $\gamma_s = 1.30 \left(\begin{array}{l} \text{value suggested by the authors in the absence of} \\ \text{experimental validation for each type of strengthening} \end{array} \right)$ $\gamma_{CFRP} = \begin{cases} 1.20 & \text{for applications with high degree of quality control} \\ 1.35 & \text{for applications with normal quality control or} \\ & \text{under difficult on-site working conditions} \end{cases}$

shear resistances (V_{teo}). Furthermore, for the concrete strength of the slabs, the values reported by the authors were considered, which are in general average strengths. The maximum shear force measured in the experimental tests (V_u) was then compared with the theoretical strength (V_{teo}).

3.1 Slabs without shear reinforcement

The literature review allowed the collection of results from 340 tests on reinforced concrete flat slabs without shear reinforcement, with symmetrical loading and with failure declared by the authors as punching shear. In order not to jeopardise the analyses, the sample space was filtered to eliminate results that are not representative of engineering practice. The criteria used to remove experimental specimens from the database were: effective depth less than 85 mm; compressive strength of concrete less than 20 MPa; flexural reinforcement with yield stress less than 300 MPa and greater than

700 MPa; omission of relevant information for calculation according to the codes. Table 2 summarises the process of collecting and assembling the database of slabs without shear reinforcement.

Table 3 presents a summary of the specimens' characteristics that effectively compose the database in the case of slabs without shear reinforcement. This final database consists of 118 samples tested by 19 different authors between 1956 and 2012. The table shows: the number of slabs per author; the size of the column side, for square columns, or the diameter, for circular columns, defined as (c); the geometry of the column cross-section, where "C" denotes columns with circular section and "S" refers to the columns with square section; the flexural reinforcement ratio (ρ); the average compressive strength of concrete reported by the authors (f_c); and the maximum shear force measured in the tests (V_u).

Figures 7 to 9 show the variation effect of some parameters on the performance of theoretical estimates of the punching shear resistance. The influence of the concrete compressive strength (f_c),

Table 2

Process to form the database with slabs without shear reinforcement

Author	N° of slabs	Slabs remaining after the filter			
		d < 85 mm	$f_c < 20$ MPa	$f_{ys} < 300$ MPa and $f_{ys} > 700$ MPa	Lack of information
Elstner and Hognestad (1956) [29]	24	24	19	17	17
Kinnunem and Nylander (1960) [30]	12	12	12	12	4
Moe (1961) [31]	13	13	13	11	5
Bernaert and Puech (1966) [32]	20	20	13	6	6
Manterola (1966) [33]	12	12	12	3	3
Yitzhaki (1966) [34]	16	0	0	0	0
Mowrer and Vanderbilt (1967) [35]	25	0	0	0	0
Schaeidt <i>et al.</i> (1970) [36]	1	1	1	1	1
Vanderbilt (1972) [37]	15	0	0	0	0
Ladner (1973) [38]	1	1	1	1	1
Marti <i>et al.</i> (1977) [39]	1	1	1	1	1
Kinnunen <i>et al.</i> (1978) [40]	8	8	8	4	0
Schaefers (1978) [41]	2	2	2	2	2
Pralong <i>et al.</i> (1979) [42]	1	1	1	1	1
Regan <i>et al.</i> (1979) [43]	10	3	3	3	0
Rankin and Long (1987) [44]	27	0	0	0	0
Regan (1986) [45]	23	13	11	11	11
Tolf (1988) [46]	8	8	8	4	4
Gardner (1990) [47]	18	9	7	0	0
Lovrovich and McLean (1990) [48]	5	0	0	0	0
Marzouk and Hussein (1991) [49]	17	14	10	10	10
Ramdane (1993) [50]	15	15	15	15	15
Tomaszewicz (1993) [51]	13	13	13	13	13
Hallgren (1996) [52]	7	7	7	7	6
Li (2000) [53]	6	6	6	6	0
Birkle and Dilger (2008) [54]	3	3	3	3	3
Guandalini <i>et al.</i> (2009) [22]	11	11	11	11	11
Sundquist and Kinnunen (2004) [55]	3	3	3	0	0
Marzouk and Hossin (2007) [56]	8	8	8	8	0
Marzouk and Rizk (2009) [57]	11	11	11	11	0
Lips <i>et al.</i> (2012) [58]	4	4	4	4	4
Slabs remaining	340	223	203	165	118

Table 3

Summary of the characteristics of the slabs in the database without shear reinforcement

Authors	N° of slabs	c (mm)	Column shape	d (mm)	ρ (%)	f_c (MPa)	V_u (kN)
Elstner and Hognestad [29]	17	254-356	S	114-121	0.5-3.7	20-51	200-578
Kinnunen and Nylander [30]	4	150-300	C	117-128	0.8-1.1	30.8-34.9	255-430
Moe [31]	5	203-305	S	114	1.1-1.5	20.8-24.5	343-433
Bernaert and Puech [32]	6	203	S	114-124	1.0-1.7	20.6-41.4	328-439
Manterola [33]	3	100-450	S	107	0.5	26.4-34.2	175-294
Schaeidt <i>et al.</i> [36]	1	500	C	240	1.3	34.9	1662
Ladner [38]	1	226	C	109	1.2	39.7	362
Marti <i>et al.</i> [39]	1	300	C	143	1.5	43.2	628
Schaefers [41]	2	120-210	C	113-170	0.6-0.8	23.1-23.3	280-460
Pralong <i>et al.</i> [42]	1	300	C	171	1.2	32.8	626
Regan [45]	11	54-250	S	93-200	0.8-1.5	29-53.3	170-825
Tolf [46]	4	250	C	197-200	0.5-0.8	28.6-31.7	444-603
Marzouk and Hussein [49]	10	150-300	S	90-120	0.7-2.1	42-80	249-645
Ramdane [50]	15	150	C	98-102	0.6-1.3	33.6-127	169-405
Tomaszewicz [51]	13	100-200	S	88-275	1.5-2.6	64.3-119.0	330-2450
Hallgren [52]	6	250	C	194-202	0.3-1.2	84.1-108.8	565-1041
Birkle and Dilger [54]	3	250-350	S	124-260	1.1-1.5	31.4-36.2	483-1046
Guandalini <i>et al.</i> [22]	11	130-260	S	96-464	0.25-1.5	27.6-40.5	118-2153
Lips <i>et al.</i> [58]	4	130-340	S	193-353	1.5-1.6	30.5-42.5	1135-2491

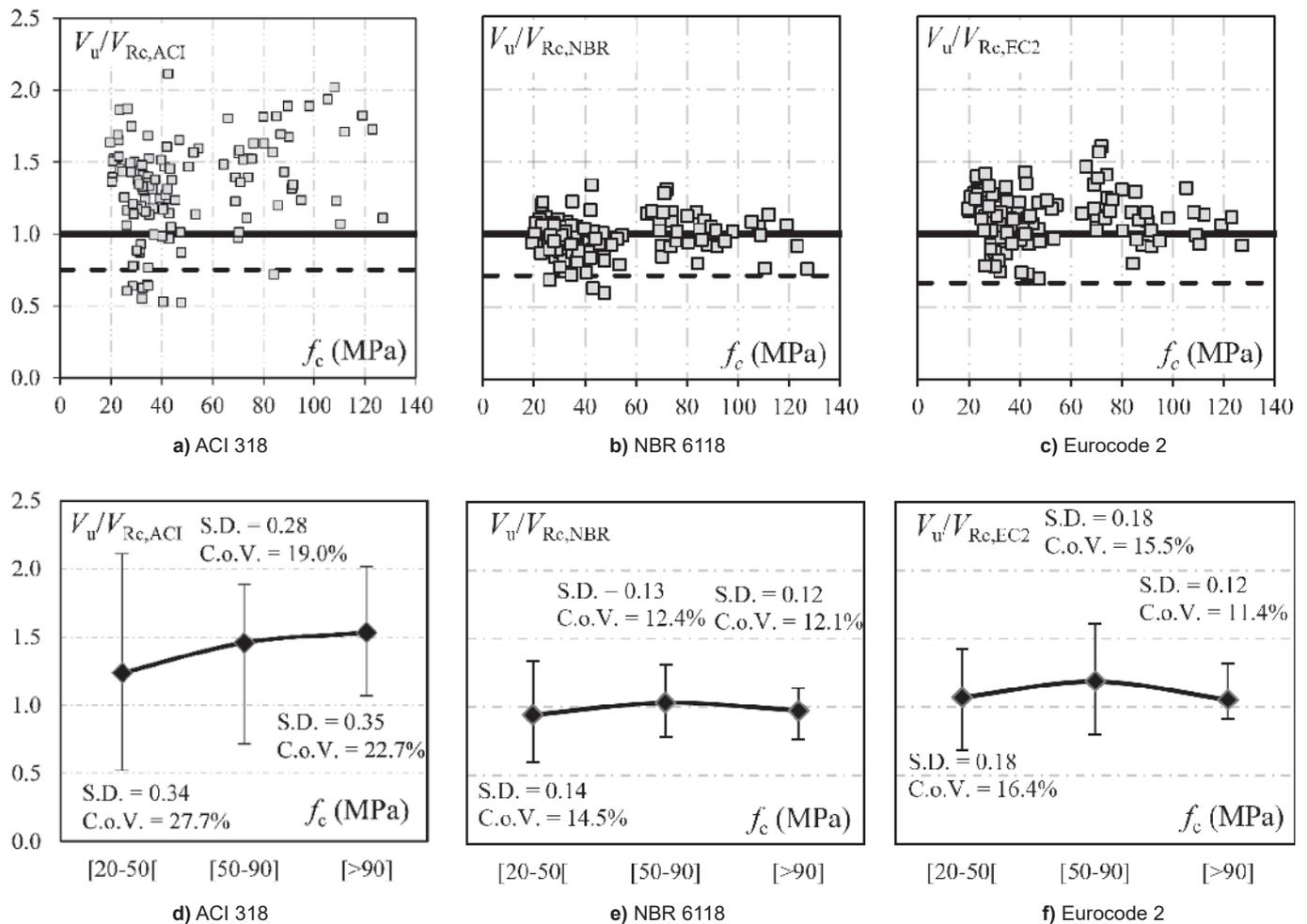


Figure 7

Influence of f_c in the theoretical prediction of resistance of slabs without shear reinforcement

the flexural reinforcement ratio (ρ) and of the effective depth of the slab (d) were evaluated

These analyzes were carried out from the distribution of the ratio between the maximum punching shear resistance (V_u) measured in the tests and the strength predicted by each code (V_{teo}). In these graphs, the solid lines represent the ideal limit, where the experimental strength would be equal to the theoretical estimate ($V_u = V_{teo}$), with the safety coefficients assumed equal to 1.0. The dashed lines represent the limit considering the theoretical resistance reduced according to the values of safety coefficients in Table 1. In parallel, in Figures 7d, 7e, 7f to 9d, 9e and 9f, analyses in three ranges of values for each parameter are performed, where it is seen the average, maximum and minimum values, standard deviation and coefficient of variation of results for each range of values analysed.

Results of Figure 7 show that ACI 318 [10] presents scattered estimates. It is notable that assuming the influence of the compressive strength of concrete on the punching shear resistance as being proportional to the square root of f_c can lead to unsafe estimates and that the limitation imposed in these equations

($f_c \leq 69$ MPa) is essential to control this trend. Regarding the influence of the compressive strength of concrete, still in Figure 7, it is possible to notice that ABNT NBR 6118 [11] presents slightly better performance than Eurocode 2 [12], what is a consequence of the limitations imposed by Eurocode 2 [12] for the consideration of the flexural reinforcement ratio (ρ) and the size effect (k).

Figure 8 shows the influence of flexural reinforcement ratio (ρ) on punching shear resistance of the tested slabs. As ACI 318 [10] ignores this parameter, it tends to underestimate the strength of slabs with values of ρ greater than 1% and to produce a significant number of unsafe predictions for slabs with ratios below 1%. It must be highlighted that for slabs with low reinforcement ratios ($\rho < 0.6\%$), both ABNT NBR 6118 [11] and Eurocode 2 [12] also present a significant number of theoretical strength predictions higher than those observed experimentally. On the other hand, for slabs with ρ greater than 2%, Figures 8e and 8f shows that, based on this database, it is not clear the need to limit the flexural reinforcement ratio as $\rho \leq 2.0$, as adopted in Eurocode 2 [12]. The effect of this limitation has left the Eurocode 2 [12] predictions, in this range, more conservative and scattered.

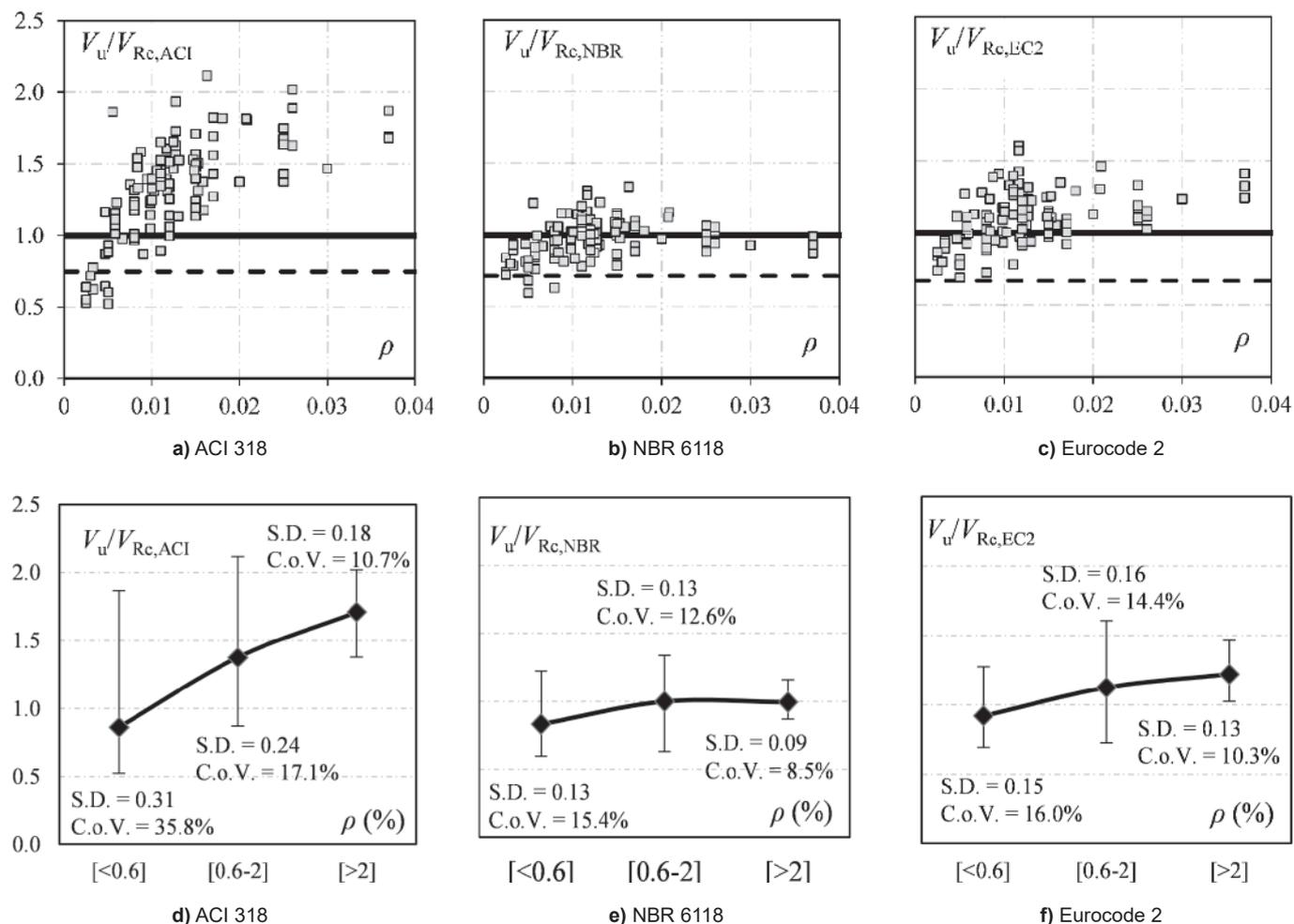


Figure 8 Influence of ρ in the theoretical prediction of resistance of slabs without shear reinforcement

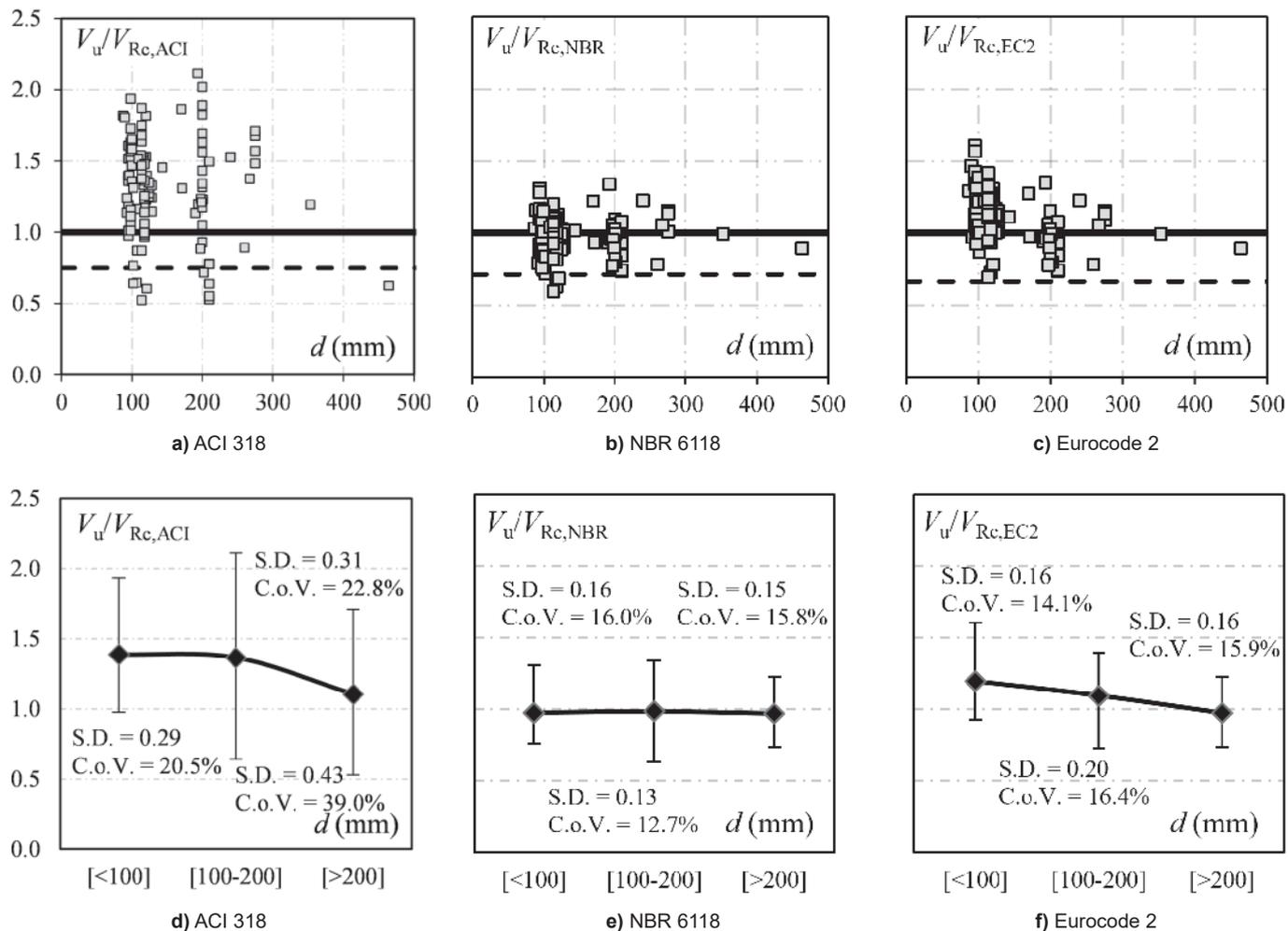


Figure 9
Influence of d in the theoretical prediction of resistance of slabs without shear reinforcement

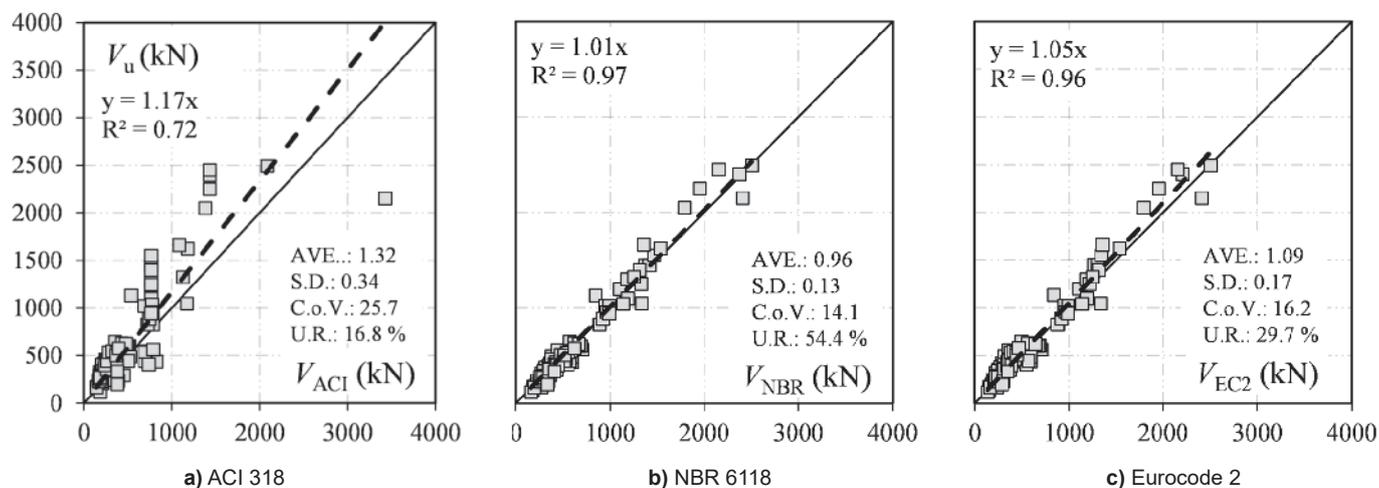
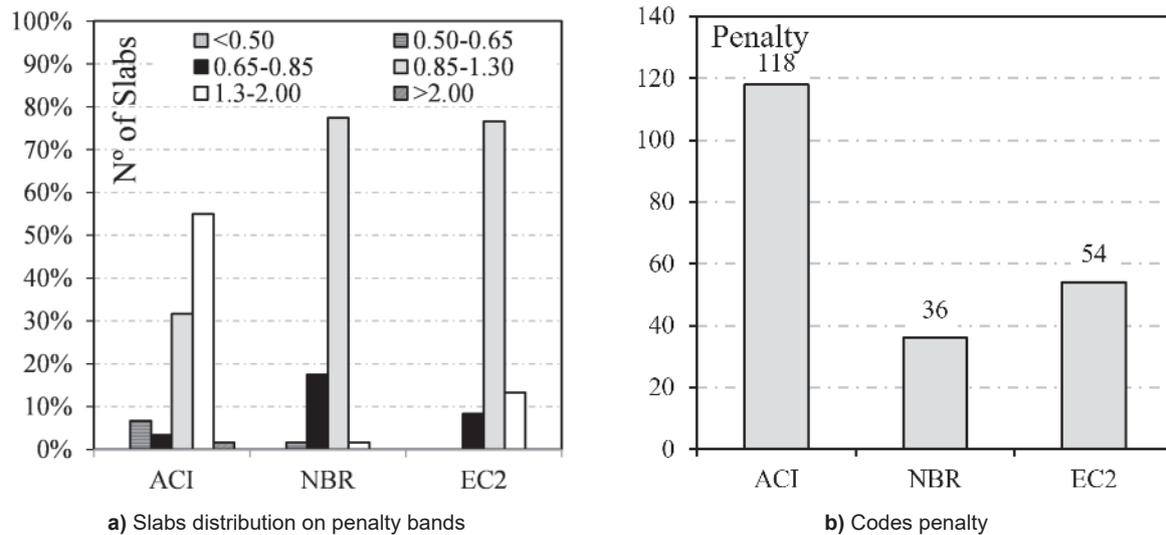


Figure 10
Accuracy of code prediction for slabs without shear reinforcement


Figure 11

Performance of codes for slabs without shear reinforcement according to Collins [23]

Figure 9 discusses the influence of the effective depth of the slabs (d) on the performance of the theoretical predictions from design codes. As ACI 318 [10] does not take into account the size effect in its equations, it shows a trend of unsafety results for slabs with an effective depth greater than 200 mm. Slab PG3 from Guandalini *et al.* [22], which combines low flexural reinforcement ratio ($\rho = 0.33\%$) and a large thickness ($d = 456$ mm), presents theoretical strength significantly higher than the one observed in the experimental test. Figure 9 also shows that ABNT NBR 6118 [11] maintains a constant average of the V_u/V_{NBR} ratio in all ranges of d , while Eurocode 2 [12] tends to underestimate the punching resistance of thin slabs ($d < 100$ mm), a consequence of the limit of $k \leq 2.0$.

Figure 10 graphically presents a general analysis of the performance of the theoretical strengths predicted by codes for slabs without shear reinforcement. This figure compares the trend line of the results (dashed line in the picture) with the ideal situation ($V_u = V_{teo}$), represented by the solid line. This figure also presents: the linear correlation coefficient of the results (R^2); the average of the results (AVE.); the coefficient of variation (C.o.V.); the standard deviation (S.D.) and percentage of unsafe results (U.R.), assumed as the cases where $V_u/V_{R,teo} < 1$. Figure 11 graphically shows the evaluation of the performance of theoretical methods, rated according to the criterion of Collins [23], called Demerit Points Classification (DPC), presented in Table 4. This classification consists of assigning a demerit scale calculated from the sum of the products of V_u/V_{teo} by the corresponding score. Table 5 presents the demerit scale proposed by Collins for V_u/V_{teo} values.

ACI 318 [10] showed the worst correlation between the experimental results and the theoretical predictions, with results of coefficient of variation equal to 25.7% and R^2 equal to 0.72. It is important to note that, despite the wide dispersion of results, ACI 318 [10] showed a low percentage (16.8%) of unsafe estimates ($V_u/V_{R,ACI} < 1$). This is due to its high average (1.32) which maintains most of its results in favour of safety. ACI 318 [10] presented 55% of its results classified, according to DPC, as conservative.

Nevertheless, 6.7% of its results are classified as dangerous, contributing to the high penalty attributed to this code. ACI 318 [10] was the most penalised code in this analysis, having the worse performance according to DPC.

The recommendations of Eurocode 2 [12] and ABNT NBR 6118 [11], which are based on CEB-FIP MC90 [24], presented similar trends regarding dispersion, with a coefficient of variation of 16.2% and 14.1%, R^2 of 0.964 and 0.970 and average of 1.10 and 0.97, respectively. It is worth mentioning that ABNT NBR 6118 [11] was the one with the best performance, with the best results of coefficient of variation, R^2 and average, being the least penalised by DPC. However, it should also be noted that 54.4% of the results were $V_u/V_{NBR} < 1.0$. As most of these values were above 0.85, this fact was ignored by DPC, which considers this as a zone of values with appropriate safety ($0.85 < V_u/V_{R,teo} \leq 1.30$).

3.2 Slabs strengthened for punching shear

A database with results of 62 experimental tests was used to evaluate the performance of the adjustments proposed in Table 1 to use ACI, EC2 and NBR 6118 for the punching shear design of slabs strengthened with post-installed steel and CFRP connectors. Table 5 presents a summary of the characteristics of the slabs used in this database. In this table, the symbology used to describe

Table 4

Demerit scale according to Collins [23]

$V_u/V_{R,teo}$	Rating	Penalty
< 0.50	Extremely dangerous	10
[0.50 – 0.65]	Dangerous	5
[0.65 – 0.85]	Low safety	2
[0.85 – 1.30]	Appropriate safety	0
[1.30 – 2.00]	Conservative	1
≥ 2.00	Extremely conservative	2

Table 5

Characteristics of the slabs strengthened against punching shear

Authors	N° slabs	Reinforcement	d (mm)	c (mm)	Column shape*	ρ (%)	f_c (MPa)	Hole/layer	N° layers	s_0 (mm)	s_r (mm)	V_u (kN)
Binici (2003) [59]	9	S	114	304	S	1.9	28	4-8	8	29	58	595-778
Binici and Bayrak (2005) [60]	2*	S	57	150	S	0.5	24	4	8	14	29	138-154
Erdogan <i>et al.</i> (2010) [17]	5	D	114	300	S	1.4	26-35	3-5	8	57-60	60-86	571-657
Erdogan <i>et al.</i> (2011) [61]	4	D	114	125-375	S and R	1.4	29-32	3	8	57	57	571-657
Rodrigues <i>et al.</i> (2015) [62]	3*	D	47	150	S	1.1	40	3-4	8	23	35	105-125
Sissakis (2007) [15]	12	S	120	85	S	1.5-2.2	27-36	6-12	6-12	30	90	550-775
Santos (2014) [16]	11	D e S	135-145	300	S	1.4-1.6	44-58	8-12	6-8	70	90	818-1185
Carvalho (2001) [25]	8	C	99-107	120	S	1.2-1.5	40-44	8	2-3	49-51	49-51	301-458
Ruiz <i>et al.</i> (2010) [18]	9	C	210	260	S	1.0-1.5	28-37	4-12	3-6	150-200	125-200	974-1690
Wörle (2014) [26]	4	C	155	300	C	2.2	36-38	8	4	59	96	612-937

* These slabs were removed from the analyses because they had a shallow effective depth.

the type of strengthening was: D and S for CFRP strengthening of types dowel and stitch; and C for strengthening with post-installed steel connectors. Table 5 also presents: the number of holes per strengthening layer; the number of strengthening layers; the distance between the first strengthening layer and the column face (s_0); and the distance between successive layers of strengthening (s_r). It should be noted the difficulty of finding experimental results of tests on slabs with post-installed shear reinforcement.

In Figures 12, 13 and 14, the results of tests where the authors inform that the failure occurred within the reinforcement region are used to discuss both the performance of the different strengthening techniques and the response of the calculation methodology presented in Table 1. The red triangles in these figures indicate

test results with post-installed steel connectors with mechanical anchorage at both ends (see Carvalho [25]). Figure 12 shows the influence of the strengthening increment, measured by the ratio between the estimated contribution from the strengthening material and the resistance of an equal slab, but without shear reinforcement ($V_{Rs,teo}/V_{Rc,teo}$), in the increase of punching shear resistance, given by the ratio between the ultimate shear force measured in the tests and the estimated punching shear resistance for the case without shear reinforcement ($V_u/V_{Rc,teo}$). The distribution of the results is confronted by a solid line showing the trend of the codes prediction for the failure within the region of the reinforcement ($V_{Rcs,teo}$) and dashed lines indicating the limitation due to crushing of the strut (V_{Rmax}).

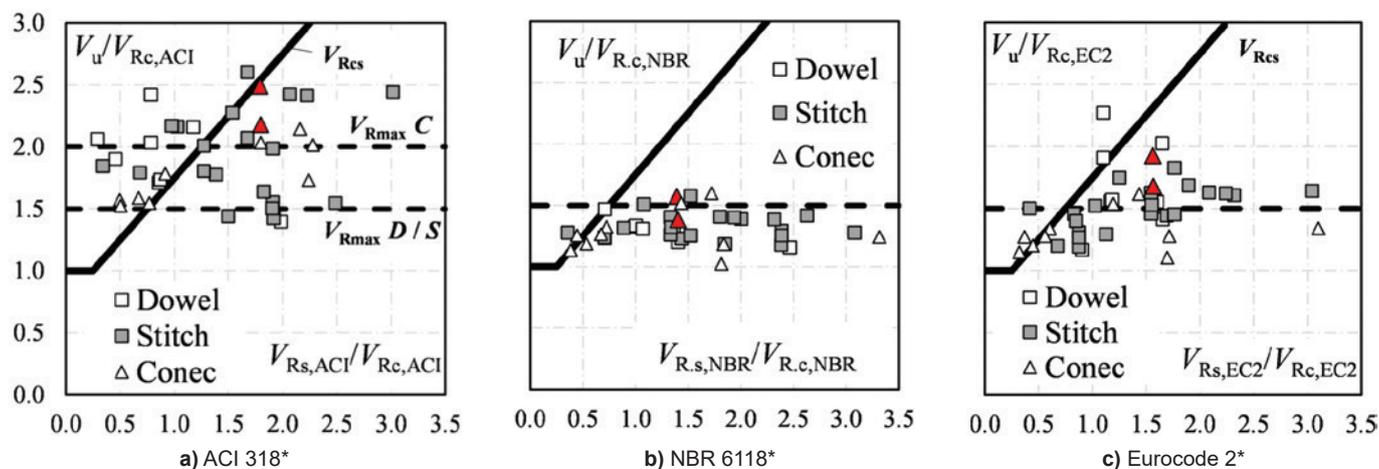


Figure 12

Performance of strengthening methods according to the proposed methodology

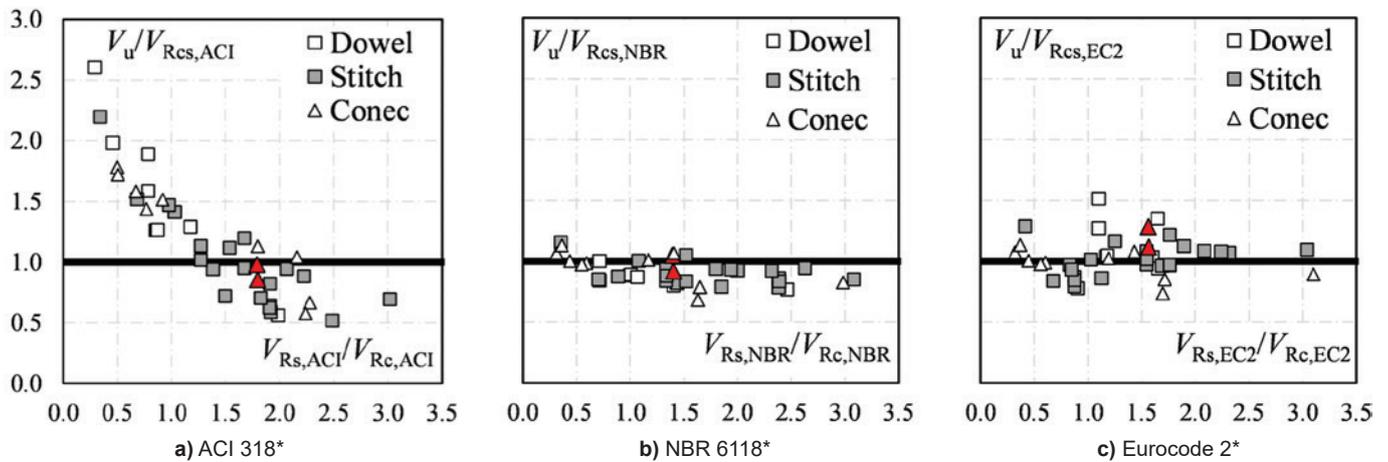


Figure 13
Influence of the increment in the shear strengthening ratio in the strength predictions for slabs failing inside the shear reinforced region

Figures 12a, 12b and 12c show that the three strengthening techniques evaluated may be efficient and have similar overall performance about their capacity of increasing punching shear resistance. In the case of methods with post-installed steel connectors, the tests of Ruiz *et al.* [18] were those that showed better performance. The authors were able to obtain increases of resistance of about 74% in comparison to the strength of the reference slab, without shear reinforcement. For all codes, the test results with steel connectors are those that show the best correlation with the trend of $V_{Rcs,teo}$, expressed by the solid line in these figures. In the case of CFRP strengthening, the tests of Santos [16] with the stitch strengthening technique were the ones that achieved better performance, showing a slightly higher performance than the dowel technique. The author achieved increases of resistance of up to 93% compared to the reference slab. In general, the tests of Sissakis and Sheikh [15] and Wörle [26] make it clear that it is fundamental

to respect the limits and the detailing rules usually recommended for pre-installed shear reinforcement to obtain an adequate performance of the strengthening method.

Figure 12a shows that the proposal for ACI 318 [10] would be the method with the highest dispersion between the theoretical results and those observed experimentally. In many cases the predictions would be very conservative, that is, with estimated resistances more than twice as low as those measured experimentally. It should also be noted that in the case of the proposal for ACI 318 [10], the small percentage of unsafe results is only guaranteed by the conservativeness of its maximum strength predictions (V_{Rmax}). Among theoretical methods, Figure 12b shows that the proposed adjustments for ABNT NBR 6118 [11] would lead to a lower dispersion between theoretical and experimental results, but the equation for V_{Rcs} , whose trend is represented by the solid line, loses correlation with the experimental basis for values of $V_{Rs} / V_{Rc} > 0.75$. For the proposed adaptation to

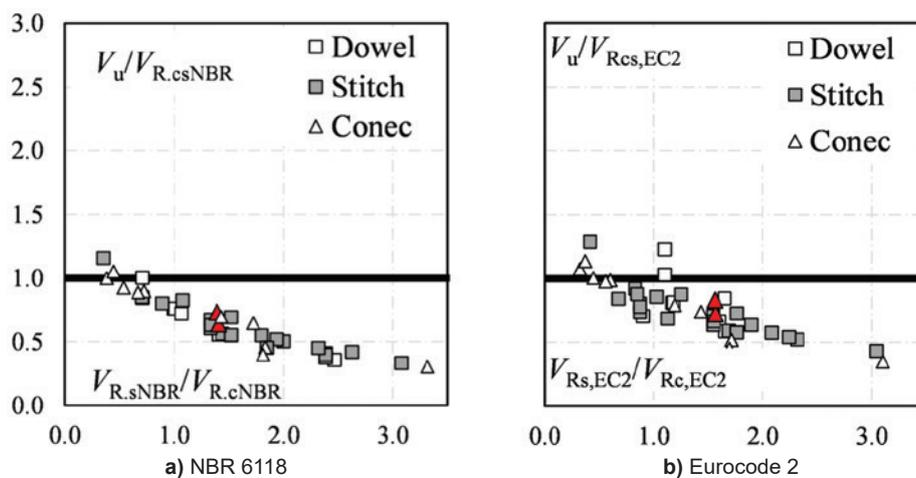


Figure 14
Performance of code provisions for failure inside the shear-reinforced region (ignoring limitations proposed in Table 1)

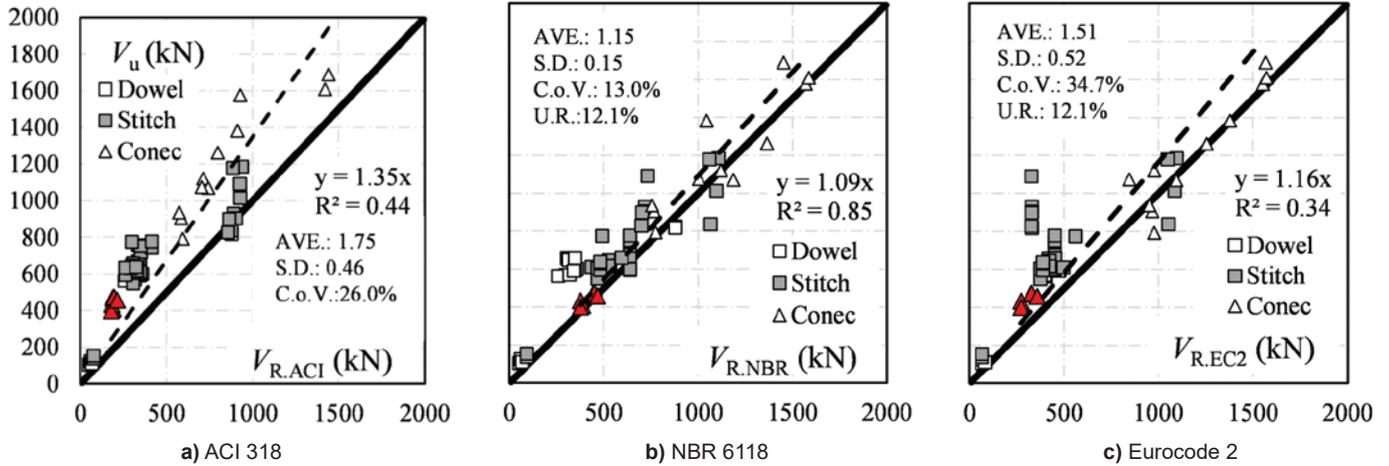


Figure 15 Accuracy of proposed adjustments for the assessment of the resistance of slabs strengthened with post-installed steel and PRFC connectors

Eurocode 2 [12] [13] [14] (see Figure 12c), it is observed that the correlation between its equation for V_{Rcs} and the database is slightly better than what was found for the adaptation of ABNT NBR 6118 [11]. It is also seen that the efficiency limitation of the strengthening in $1.5V_{Rc}$ is adequate and guarantees a good percentage of results in favour of safety.

Figure 13 shows the influence of increasing the strengthening ratio on the resistance predictions for slabs failing within the shear reinforcement region. It is observed in Figure 13a that in the case of the adaptation proposal made to ACI 318 [10], there is a tendency to underestimate the punching shear resistance in the case of slabs where the ratio $V_{Rs}/V_{Rc} < 1.0$ and to overestimate the strength in cases where $V_{Rs}/V_{Rc} > 1.5$. Figures 13b and 13c show that the strengthening efficiency limitation in $V_{Rcs} \leq 1.5V_{Rc}$, proposed to

ABNT NBR 6118 [11] and Eurocode 2, reduces or even eliminates the trend to overestimate the resistance of slabs failing within the strengthened region. Figure 14 illustrates what would be the trend of these standards if this limitation were not used.

Figure 15 presents the accuracy analysis and the statistical analysis of the proposals to verify the resistance of slabs strengthened for punching shear. Figure 16 graphically illustrates the evaluation result of these proposals according to DPC. The use of ACI 318 [10] and Eurocode 2 would lead to conservative resistance estimates. The ACI 318 [10] would perform worse than Eurocode 2 according to DPC, since it presented a large percentage of resistance estimates classified in the range of extremely conservative results. The proposed adaptation to ABNT NBR 6118 [11] showed a good correlation with the experimental basis, with average

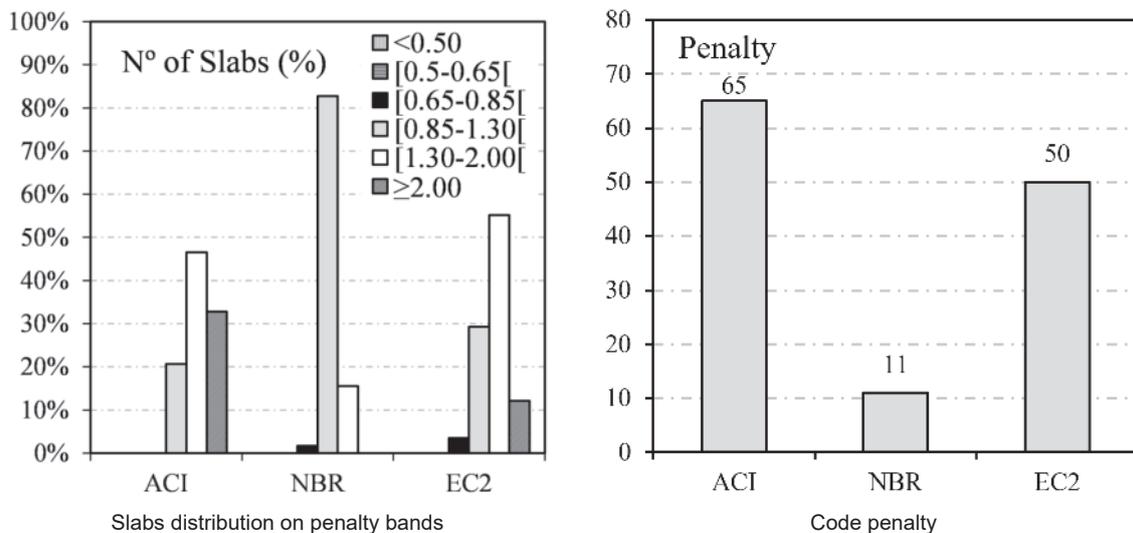


Figure 16 Performance of codes for slabs strengthened against punching according to Collins [23]

results $V_u / V_{R,NBR}$ of 1.15, the coefficient of variation of 13.0% and R^2 of 0.85, including the best performance according to the criterion of Collins [23].

4. Conclusions

This paper presented a summary of structural accidents due to punching shear failure in Brazil and abroad, and their review indicates that most of them originated from faults in the design and construction stages. This conclusion must be seen as an alert to the technical community, since design codes present recommendations that can lead to different estimates of resistance to similar situations, according to Soares and Vollum [8], among others. Also, if there is a need for strengthening, there is a lack of standardisation, both for the design and for the execution, a fact alerted by Koppitz *et al.* [9].

In the case of slabs without shear reinforcement, the analyses showed that ACI 318 [10] does not present a good correlation of its theoretical results with the trend of experimental results since it ignores essential parameters in its equations, such as the flexural reinforcement ratio and the size effect. About Eurocode 2 [12], considering this database, it was not observed any mechanical reason to justify the limitations imposed in the equations for the size effect and the flexural reinforcement ratio terms. Although they reduced the percentage of unsafe theoretical results, these limitations increased the dispersion, reducing the performance according to the criterion of Collins [23]. About the current version of the Brazilian code, a better correlation between theoretical and experimental results was observed, but with many results where the ratio between the experimental resistance (V_u) and theoretical resistance (V_{teo}) resulted in values slightly less than 1.0. As in the criterion of Collins [23] the adequate safety range is established as varying from 0.85 to 1.30, ABNT NBR 6118 [11] was the code with the best-rated performance. The analysis of the slabs strengthened for punching shear showed that the three methods evaluated can be efficient and increase the load-carrying capacity as long as the usual detailing rules are respected. About the adjustments proposed to the theoretical approaches of calculation, the proposed adaptations to ACI 318 [10] and Eurocode 2 [12] [13] [14] were the most scattered compared to the database, and their safety is guaranteed by the conservatism directly related to the recommendations for $V_{R,out}$ and $V_{R,max}$. The proposal presented for ABNT NBR 6118 [11] was the one that showed the best correlation with the database, but it was observed that it is fundamental to impose limits for the maximum performance of the strengthening, here considered as $V_{Rcs} \leq 1.5V_{Rc}$, to avoid unsafe estimates in the case of slabs failing within the shear strengthened region.

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- u_0 – length of the column perimeter
 u_1 – length of the control perimeter inside the shear–strengthened zone
 u_{out} – length of the control perimeter outside shear–strengthened zone
 A_{sw} – steel area of one layer of shear strengthening reinforcement
 C – columns with circular section
 C – post–installed steel connectors
 D – dowel strengthening
 E_{CFRP} – modulus of elasticity of CFRP
 R – rectangular column
 R^2 – coefficient of determination
 S – columns with square section
 S – stitch reinforcement
 $V_{R,c}$ – punching shear strength provided by concrete
 $V_{R,s}$ – punching shear strength provided by the strengthening reinforcement
 $V_{R,cs}$ – punching shear resistance inside the shear–strengthened zone
 $V_{R,out}$ – punching shear resistance outside the shear–strengthened zone
 $V_{R,max}$ – maximum resistance of the concrete strut close to the column
 V_{teo} – theoretical punching shear resistance
 V_u – experimental resistance

6. Notations

- ϵ_{Ru} – maximum strain of CFRP
 γ_c – safety factor for concrete material properties
 γ_s – safety factor for the material properties of reinforcing steel;
 γ_{CFRP} – safety factor for CFRP
 η_c – coefficient that accounts for the performance of strengthening systems in the punching shear resistance inside the shear–reinforced zone
 ρ – flexural reinforcement ratio
 σ_w – effective strength of transverse steel
 τ_R – stress strength
 v_{min} – minimal shear resistance
 a – major column size
 b – smallest column size
 c – size of the column
 d – effective depth of the slab
 f_c – compressive strength of concrete
 f'_c – specified compressive strength of concrete
 f_{ck} – characteristic compressive strength of concrete
 f_{yw} – yield strength of the steel connector
 k – size effect
 k_{sys} – coefficient that accounts for the performance of the strengthening system in the resistance of the concrete strut close to the column
 s_0 – clear distance from the first strengthening layer to the column side
 s_r – radial spacing between subsequent strengthening layers