Volume 8, Number 3 (June 2015) p. 260-295 • ISSN 1983-4195
http://dx.doi.org/10.1590/S1983-41952015000300003

REVISTA IBRACON DE ESTRUTURAS E MATERIAIS
IBRACON STRUCTURES AND MATERIALS JOURNAL

Influence of the column rectangularity index and of the boundary conditions in the punching resistance of slab-column connections

Influência do índice de retangularidade dos pilares e das condições de contorno na resistência à punção de ligações laje-pilar





O. S. PAIVA [a]
odilsons@gmail.com

M. P. FERREIRA [a]
mpina@ufpa.br

D. R. C. OLIVEIRA [a]
denio@ufpa.br

A. F. LIMA NETO [b]
aaraon@ufpa.br

M. R. TEIXEIRA [b]
mrt@ufpa.br

Abstract

Experimental evidence indicates that both the column rectangularity index and the boundary conditions of the connection may affect the ultimate punching resistance. This paper presents general aspects of these topics and, through the analysis of experimental results of tests on 131 slabs, evaluates the accuracy and suitability of recommendations presented by ABNT NBR 6118, Eurocode 2, ACI 318 and fib Model Code 2010. Experimental results showed that the security level of normative estimates trend to reduce as the column rectangularity increases, and in some cases, the punching resistance was overestimated. Finally, adjustments are suggested in equations presented by NBR 6118 and MC2010 in order to eliminate this trend of unsafe results.

Keywords: flat slab, punching shear, rectangular column, reinforced concrete, codes.

Resumo

Evidências experimentais indicam que tanto o índice de retangularidade dos pilares quanto as condições de contorno da ligação podem afetar a resistência última à punção. Este artigo apresenta aspectos gerais sobre estas situações e, através da análise de resultados experimentais de ensaios em 131 lajes, avalia a precisão e a adequabilidade das recomendações apresentadas pelas normas ABNT NBR 6118, Eurocode 2, ACI 318 e fib Model Code 2010. Os resultados experimentais mostraram uma tendência de redução do nível de segurança das estimativas normativas, à medida que o índice de retangularidade aumenta, chegando-se em alguns casos a superestimar a resistência à punção. Por fim, são sugeridas adaptações nas equações da norma brasileira e do MC2010 buscando eliminar esta tendência de resultados inseguros.

Palavras-chave: lajes lisas, punção, pilar retangular, concreto armado, normas.

[a] *Universidade Federal do Pará, Belém, Pará, Brasil.*
[b] *CAMTUC, Universidade Federal do Pará, Belém, Pará, Brasil.*

Received: 03 Jun 2014 • Accepted: 12 Dec 2015 • Available Online: 12 Jun 2015

1. Introduction

The design of pavements with flat slabs involves punching resistance check of the slab-column connection. This is a key stage of the project, since the structure can achieve the ultimate limit state due to the exhaustion of the shear resistant capacity in the vicinity of the slab-column connection in a brittle failure mode known as punching shear. This failure mode can lead to ruin the structure through the progressive collapse, as shown in Figure 1, which shows the partial collapse of the floor of a garage building recorded by Middleton [1], after a major earthquake in the city of Christchurch, New Zealand.

In the absence of a theory capable of explaining and accurately predict the punching shear failure mechanism, taking into account all the variables involved, the design of flat slabs is done by following design standards recommendations. These recommendations are essentially empirical and they assume a constant resistant shear stress along different control perimeters, where this resistant stress is typically estimated as a function of parameters such as the compressive strength of the concrete, effective depth of the slab, flexural reinforcement ratio, geometry and dimensions of the column. In general, the current design rules tend to ignore in its recommendations the influence of the column rectangularity index and the boundary conditions of the slab-column connection. Both can facilitate the polarization of the shear stresses, as discussed by Ferreira and Oliveira [2], which can significantly reduce the punching resistance of the slab-column connection.

This paper presents experimental results of 131 tests in models that represent the region of the slab-column connection of reinforced concrete flat slabs. Were selected cases of one-way and two-way connections supported on rectangular columns without shear reinforcement. These results are compared with theoretical estimates using the Brazilian standard, ABNT NBR 6118 [3], and also recommendations of international standards such as Eurocode 2 [4], ACI 318 [5] and fib Model Code 2010 [6, 7]. Adjustments and additional analyzes are still made, applying the treatment proposed for rectangular columns by Oliveira [8] in the expressions of Brazilian standard and MC2010 in order to assess the gains if this factor were incorporated in these rules. The reliability and accuracy of the selected standards are assessed according to a scale of demerit proposed by Collins [9].

Figure 1 – Punching shear failure of a flat slab floor (Middleton (1))

2. Literature review

2.1 Geometry and column dimensions and boundary conditions

The geometry and dimensions of the columns, as well as the boundary conditions of the slabs, can significantly influence the punching resistance of concrete flat slabs, since they affect the stress distribution in the slab-column connection. With the exception of ACI, the rules used in this article disregard the influence of these parameters, assuming a uniform distribution of stresses, since the load distribution is symmetrical, indicating that they admit the possibility that, prior to rupture, there is a significant redistribution of stresses in the slab-column connection.

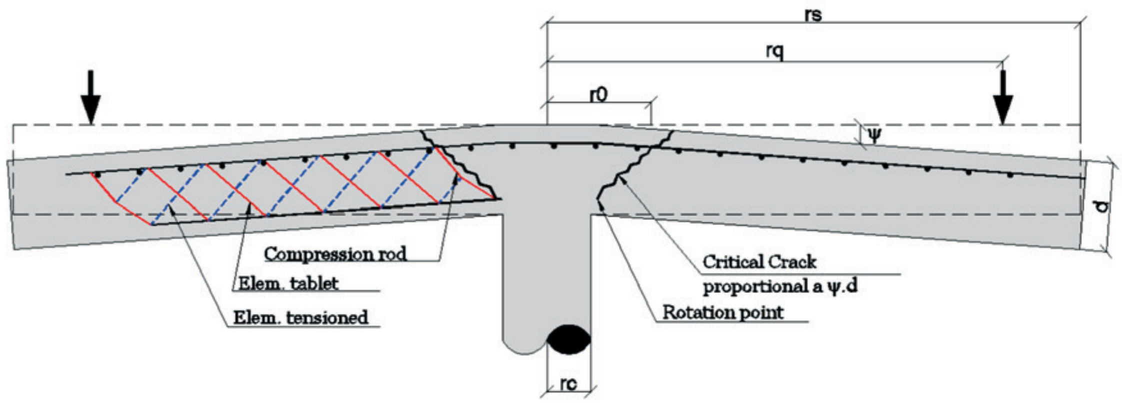
Experimental evidences as presented by Hawkins et al. [10] and Oliveira et al. [11] indicate that in the case of columns with rectangularity index greater than 2, the punching strength does not increase in direct proportion to the increase of the column section or the control perimeter length. This behavior is explained due to the polarization of shear stresses around the corners of the rectangular columns, which can lead to premature punching failures. This can be especially dangerous for the design of buildings with flat slabs because, in practice, it is common to have columns of buildings with rectangularity index 4-5 at least.

The boundary conditions may also change the distribution of shear stresses around the column. One-way slab panels trend to concentrate the shear stresses on the column faces perpendicular to the smaller span. This can be easily checke by linear-elastic computational analysis, commonly used in the design of buildings with flat slabs. In these cases, if the connection has columns with high rectangularity index, this can further enhance the polarization stresses, reducing the punching resistance of the slab-column connection. No standard presents recommendations to help structural designers in these cases.

2.2 Critical shear crack theory (CSCT)

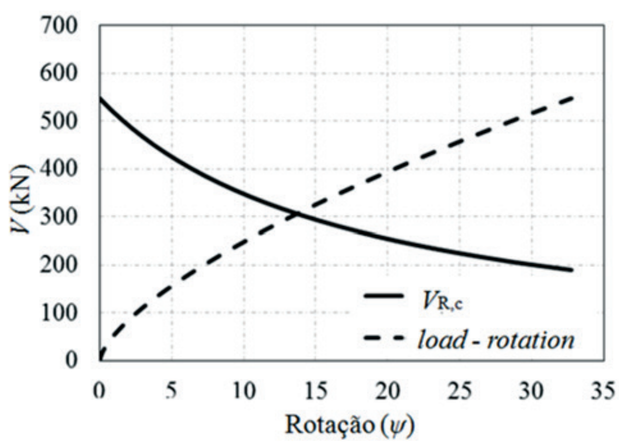
The punching resistance is a subject that has always received much attention, with numerous previous studies seeking to understand the failure mechanism and how it may be affected by the different variables involved in the practice of the structural design of flat slabs. Recently, fib Model Code 2010 [6, 7] introduced new formulations, based on the Critical Shear Crack Theory (CSCT). This theory was initially developed by Muttoni and Schwartz [12], but has been enhanced in works such as Muttoni [13] Fernandez Ruiz and Muttoni [14] and Sagaseta et al. [15]. It has as fundamental hipothesys that the punching strength decreases with the increasing of rotation of the slab due to the appearance of a critical shear crack, which propagates along the slab thickness, cutting the concrete strut that transmits the shear force to the column.

Figure 2 – Critical shear crack theory (adapted from Moraes Neto (36))



The opening of the critical shear crack, which is assumed as proportional to the product ψ.*d* (see Figure 2), reduces the strength of the concrete strut and can lead to the punching failure. The shear transmission in the critical shear crack is assumed to be a function of the surface roughness, which is directly related to the maximum size of coarse aggregate. These concepts led to the development of Equation 1, which defines the punching resistance for a slab without shear reinforcement, expressed as a function of: the length of the control perimeter (u_1) taken *d/2* away from the column face; the effective depth of the slab (*d*); the compressive strength of concrete (f_c); the product ψ.*d*, where ψ is the rotation of the slab, calculated according to Equation 2; the maximum aggregate size (d_g); and a reference aggregate size (d_{g0}), assumed to be 16 mm. Using the equations 1 and 2 it is possible to make the graph shown in Figure 3, where the punching resistance of the connection is determined at the point of intersection of the two curves.

Figure 3 – Graphic representation of the punching strength determination according to CSCT



$$V_{R,c} = \frac{3}{4} \cdot \frac{u_1 \cdot d \cdot \sqrt{f_c}}{1 + 15 \cdot \dfrac{\psi \cdot d}{d_{g0} + d_g}} \quad (1)$$

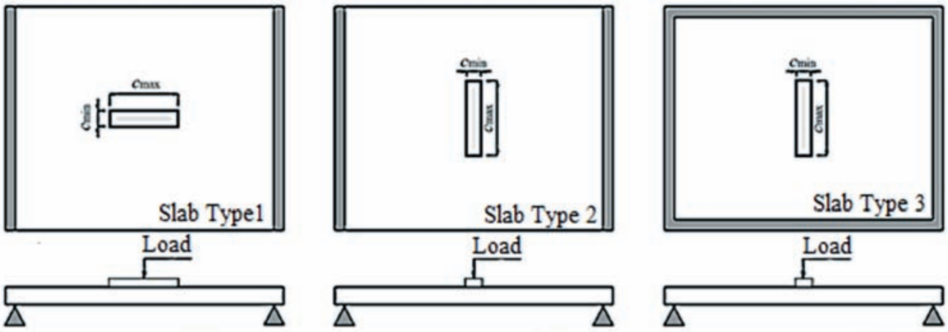
$$\psi = 1,5 \cdot \frac{r_s}{d} \cdot \frac{f_{ys}}{E_s} \cdot \left(\frac{V_E}{V_{flex}} \right)^{3/2} \quad (2)$$

Where:
r_s is the distance between column of slab and line of contraflexure of moments;
f_{ys} is the yield strength of flexural reinforcemen;
E_s is the modulus of elasticity of the flexural reinforcement;
V_E is the applied force;
V_{flex} is the flexural strength of the slab, calculated by the yield lines theory.

2.3 Flexural factors (λ)

Oliveira [8] proposed a parameter λ to correct the punching resistance estimates for the case of rectangular slabs supported on columns in order to take into account both the flexural behavior of the slabs (boundary conditions) and the rectangularity index and the orientation of the columns. They were developed with reference to experimental results of tests on reinforced concrete slabs under point load.
The bending factors (λ) were initially proposed for correction of the theoretical results from the former CEB-FIP MC90 [16], which are still the basis of the recommendations presented by the Brazilian code for the design of concrete structures. In the proposed methodology, the slabs were classified into three groups. For each group,

Figure 4 – Bending factors for flat slabs (adapted from Oliveira (8))



$$\lambda = \left(\frac{C_{máx}}{d}\right)^{0.17} \qquad \lambda = 0.93.\left(\frac{C_{máx}}{d}\right)^{0.14} \qquad \lambda = 1.03.\left(\frac{C_{máx}}{d}\right)^{0.02}$$

a value of the flexural factor (λ) was proposed for the correction of the theoretical punching resistance, as shown in Figure 4.

In the proposal presented by Oliveira et al. [11], the flexural factors function as a constant to correct the parameter of 0,18, which is the characteristic value if the original equation from MC90, resulting in Equation 3 . The results of this change were significantly better compared with the experimental results and eliminated the tendency to overestimate the punching resistance of flat slabs supported on rectangular columns. Similarly, this methodology will be used in the estimates made with the recommendations from NBR 6118 and fib Model Code 2010, in order to assess the gains from the implementation of this parameter.

$$V_{R,c} = \frac{0,18}{\lambda}\cdot\left(1+\sqrt{\frac{200}{d}}\right)\cdot\left(100.\rho.f_c^{'}\right)^{1/3}\cdot u\cdot d \qquad (3)$$

2.4 Demerit points classification

Collins [9] presented a scale to classify the reliability of code's provisions, denominated as Demerit Points Classification (*DPC*), which takes into account the safety, accuracy and scattering as a function of the ratio between the ultimate resistance observed in experimental tests (V_{exp}), and the estimated theoretical load capacity (V_{teo}). Table 1 shows an adaptation made in this research to the original values proposed by Collins. Thus, depending on the range of the results of V_{exp}/V_{teo}, penalties are stablished to the standard code under analysis. The value of the demerit of each code is calculated by the sum of the products of the number of slabs in each interval, for their corresponding penalty. The higher the value of the total sum is, the worse is considered the code provision.

Table 1 – Classification by demerit points (Collins (9))

V_{exp}/V_{teo}	Classification	Demerit points
<0,50	Extremely dangerous	10
(0,50-0,85)	Dangerous	5
(0,85-1,15)	Appropriat and safe	0
(1,15-2,00)	Conservative	1
≥2,00	Extremely conservative	2

2.5 Code provisions

2.5.1 NBR 6118

The NBR 6118 [3] assumes that the punching resistance of reinforced concrete flat slabs without shear reinforcement shall be checked in two regions: the resistance to diagonal tension should be checked at the control perimeter u_1, according to Equation 4; the maximum resistance (crushing of the concrete strut close to the column) must be checked in the perimeter u_0 around the column using Equation 5. Figure 5a shows details of the control perimeters proposed by the Brazilian standard.

$$V_{R,c} = 0,18\cdot\left(1+\sqrt{200/d}\right)\cdot\left(100\cdot\rho\cdot f_c\right)^{1/3}\cdot u_1\cdot d \qquad (4)$$

Where:

ρ is the ratio of flexural reinforcement, calculated as $\rho = \sqrt{\rho_x \cdot \rho_y}$

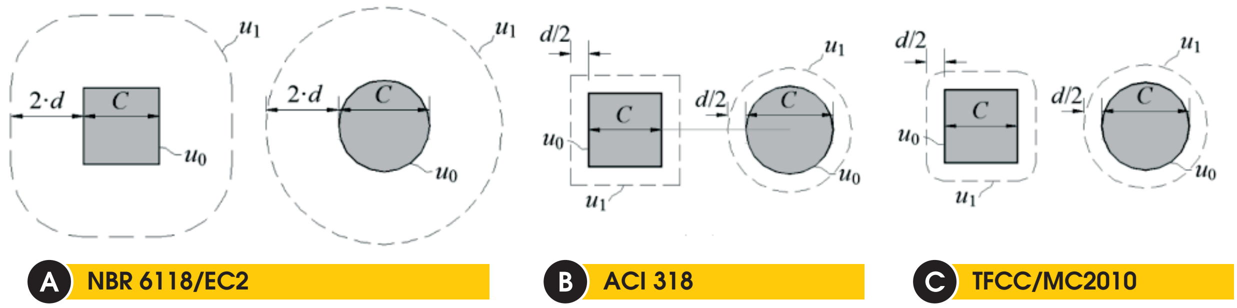
ρ_x and ρ_y are the ratios of flexural reinforcement in two orthogonal directions, calculated assuming a width equal to the size of the column plus 3*d* to either side;

f_c is the compressive strength of concrete in MPa ($f_c \leq 50$ MPa);

u_1 is the control perimeter taken at a distance of 2.*d* away from the column face;

d is the effective depth of the slab in mm.

Figure 5 – Control perimeter for different calculation methods



$$V_{R,\max} = 0{,}27 \cdot \alpha_{v1} \cdot f_c \cdot u_0 \cdot d \quad (5)$$

Where:
$\alpha_{v1} = (1 - f_c/250)$
u_0 is the perimeter of the column.

2.5.2 Eurocode 2

The recommendations made by Eurocode 2 [4] are similar to those given by the Brazilian standard, since both are based on the recommendations of CEB-FIP MC90 [16]. It differs from the Brazilian standard by imposing limits on the value of the Size effect ($\xi \le 2{,}0$) and flexural reinforcement ratio ($\rho \le 2{,}0\%$). This was done trying to eliminate the trend of unsafe results. This has been discussed in other researches like those from Sacramento et al. [17] and Oliveira et al. [18]. Therefore, the punching resistance is assumed as the lowest value given by Equations 6 and 7. The control perimeters are equal to those in the Brazilian code (see Figure 5a).

$$V_{R,c} = 0{,}18 \cdot \xi \cdot (100.\rho.f_c)^{1/3} \cdot u_1 \cdot d \quad (6)$$

$$V_{R,max} = 0{,}3 \cdot f_c \cdot \left(1 - \frac{f_c}{250}\right) \cdot u_0 \cdot d \quad (7)$$

Where:
f_c is the compressive strength of concrete in MPa ($f_c \le 90$ MPa);
ρ is the ratio of flexural reinforcement of the slab, obtained as $\rho = \sqrt{\rho_x \cdot \rho_y} \le 2{,}0\%$;
ρ_x and ρ_y are the rates in the x and y directions, respectively. The bars are to be considered within a region far from the 3.*d* sides of the column.

$\xi = 1 + \sqrt{\frac{200}{d}} \le 2{,}0$ is a dimensionless number and *d* is expressed in mm.

2.5.3 ACI 318

ACI 318 [5] recommends that the punching strength in reinforced concrete flat slabs without shear reinforcement shall be checked through the analysis of the shear stresses in a control perimeter taken at a distance equal to *d*/2 from the column face or the edges of the loaded area, as shown in Figure 5b using Equation 8.

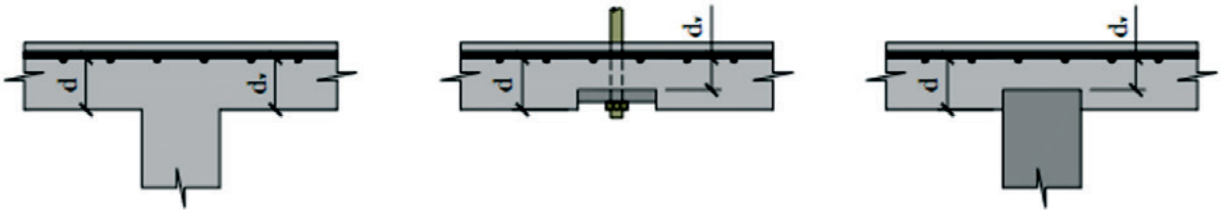
$$V_{R,c} = \min \begin{cases} \left(1 + \frac{21}{\beta_c}\right) \cdot \frac{-}{6} \cdot \sqrt{f_c} \cdot u_1 \cdot d \\ \left(\frac{\alpha_s \cdot d}{u_1} + 2\right) \cdot \frac{1}{12} \cdot \sqrt{f_c} \cdot u_1 \cdot d \\ \frac{1}{3} \cdot \sqrt{f_c} \cdot u_1 \cdot d \end{cases} \quad (8)$$

Where:
β_c is the ratio of long side to short side of thecolumn;
α_s is a constant taken as 40 for interior columns, 30 for edge columns and 20 for corner columns;
u_1 the length of a control perimeter taken *d*/2 away from the column face;
f_c is the compressive strength of concrete in MPa ($f_c \le 69$ MPa).

2.5.4 fib model code 2010

The recommendations presented by the fib Model Code 2010 [6, 7] are based on a physical model fundamented on the Critical Shear Crack Theory, presented briefly in section 2.2 of this paper. In this code, the punching resistance should be checked around a control perimeter (b_0), admitted at a distance of 0,5.d_v from the column faces or edges of the loaded area with the geometry as shown in Figure 5c. The effective depth of the slab, d_v, should consider

Figure 6 – Effective depth of the slab (fib Model Code 2010 (6, 7))



the level of support, as shown in Figure 6. In the case of flat slabs without shear reinforcement, the punching resistance shall be calculated according to Equation 9.

$$V_{R,c} = k_{\psi} \cdot \frac{\sqrt{f_c}}{\gamma_c} \cdot b_o \cdot d_v \qquad (9)$$

Where:
f_c is the compressive strength of concrete;
d_v is the effective depth of the slab;
γ_c is the concrete safety factor with a value of 1.5. For purposes of the calculations it was assumed asequal to 1.
k_{ψ} is calculated by Equation 10 and depends on the slab rotation in the support region.

$$k_{\psi} = \frac{1}{1{,}5 + 0{,}9 \cdot \psi \cdot d \cdot k_{dg}} \leq 0{,}6 \qquad (10)$$

$$k_{dg} = \frac{32}{16 + d_g} \geq 0{,}75 \qquad (11)$$

Where:
d_g is the maximum diameter of the aggregate used in the concrete slab.
fib Model Code 2010 states that the rotation of the slab (ψ) can be calculated with different levels of approximation, depending on the accuracy required. In this case, the precision obtained increases as a function of the complexity of the analysis. The level I of approximation is recommended for the primary design of new structures. The level II is suitable for both the design of new structures and for checking of existing structures. In special cases, where the characteristics of the strucuture or of the loads are nonconventional, the level III of approximation is recommended in order to better estimate the response of the slab. In special cases, it is permitted that the rotation is obtained using non-linear analysis, corresponding to the level IV of approximation.
In the practice of the design of concrete structures, level I can be used, for example, in the case of regular flat slabs designed according to an elastic analysis without significant redistribution of internal forces, and the rotation can be estimated according to Equation 12. In cases where significant moment redistribution is considered in design, Equation 13 can be used to estimate the slab rotation, referring to the level II of approaximation.

$$\psi = 1{,}5 \cdot \frac{r_s}{d} \cdot \frac{f_{ys}}{E_s} \qquad (12)$$

$$\psi = 1{,}5 \cdot \frac{r_s}{d} \cdot \frac{f_{ys}}{E_s} \cdot \left(\frac{m_s}{m_R}\right)^{1{,}5} \qquad (13)$$

Where:
f_{ys} is the yield strength of the flexural reinforcement;
E_s is the modulus of elasticity of the flexural reinforcement;
r_s is the position where the radial bending moment is zero with respect to the support axis. The value of r_s can be approximated as 0.22 L_x or 0.22 L_y for the x and y directions, respectively, for regular flat slabs where the ratio of the spans (L_x/L_y) is between 0.5 and 2.0.
m_s is the average moment per unit length applied in the slab-column connection;
m_R is the flexural strength per unit length of the slab-column connection.
Both moments are calculated for a strip with width equal to $b_s = 1{,}5 \cdot (r_{s,x} \cdot r_{s,y})^{0{,}5} \leq L_{min}$. In this case, r_{sx} and r_{sy} denotes the point at which the moments are equal to zero, having as reference the axis of the support in *x* and *y* directions. The approximate value of m_{Sd} depends on the location of the column in the building. The code considers three possible locations for the columns: (1) internal to the building, (2) at the edge; and (3) at the corner. In the case of internal columns, where the slab has equal flexural strength in both directions, m_s is calculated by the simplified expression presented in Equation 14. The average flexural strength per unit length (m_R) can be obtained according to Equation 15.

$$m_s = \frac{V_E}{8} \qquad (14)$$

$$m_R = \rho \cdot d^2 \cdot f_{ys} \cdot \left(1 - \frac{\rho \cdot f_{ys}}{2 \cdot f_c}\right) \tag{15}$$

Where:
ρ is the flexural reinforcement ratio.
In the level III of approximation, the coefficient 1.5 in Equation 13 may be replaced by 1,2 if the values of r_s and m_s are calculated by a linear-elastic model. In the level IV of approaximation, the rotation (ψ) of the slab-column connection is obtained through a non-linear analysis, accounting for cracking, tension-stiffening effects, yielding of the reinforcement and any other non-linear effects relevant for providing an accurate assessment of the structure.

3. Database

In order to evaluate and compare the code provisions presented, a database comprising experimental results of tests in reinforced concrete flat slabs without shear reinforcement was used. This database has results of one-way and two way slabs

Table 2 – Characteristics of the slabs in the database

Author	Slab	Type	d (mm)	c_{min} (mm)	c_{max} (mm)	ρ (%)	f'_c (MPa)	f_{ys} (MPa)	E_{sf} (GPa)	dg (mm)	P_{flex} (kN)	V_{exp} (kN)	Failure mode
Ferreira [30]	L1a	2	87,0	85	85	0,94	42,4	488	220,0	12,0	255,1	174,0	P
	L1b	2	89,0	85	85	1,18	51,4	488	220,0	12,0	264,2	231,5	FP
	L1c	2	87,0	85	85	1,48	43,5	488	220,0	12,0	255,8	190,0	P
Lima Neto [28]	AL1	1	100,0	150	170	1,37	42,0	616	207,0	19,0	472,0	300,0	P
	AL2	1	102,0	150	170	1,34	44,0	616	207,0	19,0	485,0	380,0	P
	AL3	1	100,0	150	170	1,37	41,0	616	207,0	19,0	472,0	340,0	P
	AL4	1	95,0	150	170	1,44	47,0	616	207,0	19,0	452,0	310,0	P
Oliveira [8]	L1a	1	107,0	120	120	1,09	57,0	750	234,0	15,0	241,2	234,0	FP
	L1b	1	108,0	120	120	1,08	59,0	750	234,0	15,0	657,1	322,0	P
	L1c	3	107,0	120	120	1,09	59,0	750	234,0	15,0	706,7	318,0	P
	L2a	1	109,0	120	240	1,07	58,0	750	234,0	15,0	261,7	246,0	FP
	L2b	2	106,0	120	240	1,1	58,0	750	234,0	15,0	644,6	361,0	P
	L2c	3	107,0	120	240	1,09	57,0	750	234,0	15,0	735,6	331,0	P
	L3a	1	108,0	120	360	1,08	56,0	750	234,0	15,0	277,0	241,0	FP
	L3b	2	107,0	120	360	1,09	60,0	750	234,0	15,0	645,2	400,0	P
	L3c	3	106,0	120	360	1,1	54,0	750	234,0	15,0	745,8	358,0	P
	L4a	1	108,0	120	480	1,08	56,0	750	234,0	15,0	295,3	251,0	FP
	L4b	2	106,0	120	480	1,1	54,0	750	234,0	15,0	637,1	395,0	P
	L4c	3	107,0	120	480	1,09	56,0	750	234,0	15,0	792,2	404,0	P
	L5a	1	108,0	120	600	1,08	57,0	750	234,0	15,0	318,9	287,0	FP
	L5b	2	108,0	120	600	1,08	67,0	750	234,0	15,0	655,4	426,0	P
	L5c	3	109,0	120	600	1,07	63,0	750	234,0	15,0	857,7	446,0	P
Hawkins et al. [10]	1	1	117,3	305	305	1,12	30,9	419	200,0	12,0	362,0	391,0	F
	2	1	117,3	203	406	1,12	26,9	419	200,0	12,0	384,9	358,0	P
	3	1	117,3	152	457	1,12	32,6	419	200,0	12,0	400,0	340,0	P
	4	1	117,3	114	495	1,12	31,6	419	200,0	12,0	411,0	337,0	P
	5	2	117,3	152	457	1,12	27,4	419	200,0	12,0	489,2	362,0	P
	6	2	117,3	152	457	1,12	23,1	419	200,0	12,0	322,6	342,0	F
	7	3	117,3	152	457	0,86	26,4	419	200,0	12,0	417,9	326,0	P
	8	3	120,7	114	495	0,8	26,6	422	200,0	12,0	416,9	321,0	P
	9	3	120,7	152	305	0,76	30,1	422	200,0	12,0	350,0	322,0	P
Regan e Rezai-Jorabi [24]	14R	1	79,0	75	100	1,54	31,0	670	200,0	13,0	236,9	154,0	P
	15R	2	79,0	100	150	1,54	30,8	670	200,0	13,0	235,6	172,0	P
	19R	2	79,0	100	150	1,51	29,0	670	200,0	13,0	288,1	170,0	P

P is the failure mode by punching; FP is the failure mode by ductile punching; and F is the failure mode by bending.

supported on square and rectangular columns, with a total of 131 results presented by Forssel and Holmberg [19], Elstner and Hognestad [20], Mowrer and Vanderbilt [21], Hawkins et. al. [10], Regan [22], Regan [23], Regan and Pray-Jorabi [24], Tomaszewicz [25], and Teng Leong [26], Borges [27], Lima Neto [28], Oliveira [8], Al-Yousif and Regan [29], Ferreira [30], Vilhena et al. [31], Carvalho [32], Moorish [33], Damasceno [34] and Moraes Neto [35]. In the case of the fib predictions, as some authors did not specify the maximum diameter of the aggregates, an average value of 13 mm was assumed. Table 2 shows the characteristics of the slabs.

As the fib Model Code 2010 does not present neither instructions to estimate the punching strength of slabs supported on rectangular columns, nor provisions to treat the cases where the slab panels have different spans (rectangular for example), it was necessary to make some assumptions in order to allow its use in these analyses. In the case of rectangular panels, the rotations were calculated in both directions and the higher values were used to estimate the punching resistance. The flexural resistance (V_{flex}) provided by the authors of the tests was used to calculate the rotations of the slabs. Finally, in case of rectangular columns, a rectangular control perimeter was assumed, taken at a distance of $0{,}5.d_v$ from the column faces.

In order to improve the provision presented by NBR 6118 [3], the bending factores proposed by Oliveira [8] were incorporated in order to check the accuracy of its theoretical estimates. In the case of the Brazilian code, it was also cheched the effect of reducing the coefficient of 0.18 to 0.16 in Equation 4, as suggested by Sacramento et al. [17]. The bending factores suggested by Oliveira were also applied in the recommendations presented by fib Model Code 2010, in order to check the possible improvements.

Table 2 presents the failure mode of the slabs, based preferably on the experimental observations presented by the authors. In cases where the author does not present this information, the failure mode was established through the ratio between the ultimate resistance observed in the test and the estimated flexural resistance. For 1,10 the failure mode was assumed as flexure. For Ø<0,90 the failure mode was assumed as punching shear. In intermediate cases (0,90 <Ø <1,10), the failure mode was considered as ductile punching shear. These results are presented and discussed and

Table 2 – (cont. 1)

Author	Slab	Type	d (mm)	c_{min} (mm)	c_{max} (mm)	ρ (%)	f'_c (MPa)	f_{ys} (MPa)	E_{sf} (GPa)	dg (mm)	P_{flex} (kN)	V_{exp} (kN)	Failure mode
Teng et al. [26]	OC11	3	105,3	200	200	1,81	36,0	452	200,0	13,0	604,3	423,0	P
	OC13	3	107,3	200	600	1,71	35,8	452	200,0	13,0	676,2	568,0	P
	OC15	3	102,8	200	1000	1,76	40,2	452	200,0	13,0	697,8	649,0	P
	OC13	3	109,8	200	600	1,67	33,0	470	200,0	13,0	715,5	508,0	P
	C11F22	3	155,0	250	250	1,72	35,4	460	200,0	13,0	1306,3	627,0	P
	C13F22	3	155,0	250	750	1,66	35,6	460	200,0	13,0	1494,3	792,0	P
	C15F22	3	160,0	250	1250	1,64	35,4	460	200,0	13,0	1760,0	1056,0	P
	C13F11	3	159,0	250	750	1,07	35,5	520	200,0	13,0	1183,1	769,0	P
Forssel e Holmberg [19]	10	3	104,0	25	300	0,68	17,6	500	200,0	13,0	221,4	186,0	P
	11	3	112,0	140	540	0,63	17,6	500	200,0	13,0	281,8	279,0	P
	12	3	108,0	140	340	0,65	17,6	500	200,0	13,0	308,1	265,0	P
Elstner e Hognestad [20]	A7	1	114,5	254	254	2,48	28,5	321	200,0	13,0	416,7	400,0	P
	A8	1	114,5	356	356	2,48	21,9	321	200,0	13,0	423,3	436,0	P
	A2a	3	114,5	254	254	2,48	13,7	321	200,0	13,0	586,0	334,0	P
	A2b	3	114,5	254	254	2,48	19,5	321	200,0	13,0	655,7	400,0	P
	A2c	3	114,5	254	254	2,48	37,4	321	200,0	13,0	741,3	467,0	P
	A7b	3	114,5	254	254	2,48	27,9	321	200,0	13,0	711,1	512,0	P
	A5	3	114,5	356	356	2,48	27,8	321	200,0	13,0	762,9	534,0	P
Regan [22]	DT1	1	190,0	150	150	1,28	43,6	530	200,0	13,0	847,8	780,0	P
	BD2	1	101,0	100	100	1,28	42,2	530	200,0	13,0	299,0	293,0	P
Al-Yousif e Regan [29]	1	1	80,0	100	500	0,98	23,6	472	200,0	13,0	229,6	163,0	P
	2	3	80,0	100	500	0,98	23,2	472	200,0	13,0	243,0	209,0	P
	3	2	80,0	100	500	0,98	21,2	472	200,0	13,0	225,0	189,0	P
	4	3	80,0	300	300	0,98	22,0	472	200,0	13,0	239,6	242,0	P
Carvalho [32]	1	2	85,0	85	85	1,32	52,0	530	646,0	19,0	220,2	185,0	P
	2	2	86,0	85	255	1,32	52,0	530	646,0	19,0	223,8	226,0	P
	3	2	85,0	85	425	1,32	50,0	530	646,0	19,0	219,3	239,0	P

P is the failure mode by punching; FP is the failure mode by ductile-punching; and F is the failure mode by bending.

Table 2 – (cont. 2)

Author	Slab	Type	d (mm)	c_{min} (mm)	c_{max} (mm)	ρ (%)	f'_c (MPa)	f_{ys} (MPa)	E_{sf} (GPa)	dg (mm)	P_{flex} (kN)	V_{exp} (kN)	Failure mode
Borges [27]	L42	3	139,0	200	400	1,46	43,2	604	200,0	13,0	1152,6	657,0	P
	L42a	3	164,0	200	400	1,23	36,2	604	200,0	13,0	1358,8	693,0	P
	L45	3	154,0	200	600	1,31	42,0	604	200,0	13,0	1352,5	798,0	P
	L46	3	164,0	200	800	1,23	39,3	604	200,0	13,0	1518,3	911,0	P
	L41	3	139,0	150	250	1,46	44,7	604	200,0	13,0	1103,9	563,0	P
	L41a	3	164,0	150	250	1,23	38,9	604	200,0	13,0	1304,3	600,0	P
	L43	3	164,0	150	450	1,23	38,7	604	200,0	13,0	1369,8	726,0	P
	L44	3	164,0	150	600	1,23	40,0	604	200,0	13,0	1435,8	761,0	P
Mowrer e Vanderbilt [21]	1	3AL	51,0	102	102	1,1	28,6	386	200,0	13,0	75,4	86,0	F
	2	3AL	51,0	102	102	2,2	24,9	386	200,0	13,0	136,0	102,0	P
	3	3AL	51,0	152	152	1,1	21,1	386	200,0	13,0	77,5	79,0	F
	4	3AL	51,0	152	152	2,2	18,0	386	200,0	13,0	132,0	99,0	P
	5	3AL	51,0	203	203	1,1	15,5	386	200,0	13,0	78,8	93,0	F
	6	3AL	51,0	203	203	2,2	27,2	386	200,0	13,0	154,7	133,0	P
	7	3AL	51,0	254	254	1,1	23,3	386	200,0	13,0	87,9	109,0	F
	8	3AL	51,0	254	254	2,2	22,9	386	200,0	13,0	158,3	152,0	P
	9	3AL	51,0	305	305	1,1	28,0	386	200,0	13,0	95,2	119,0	F
	10	3AL	51,0	305	305	2,2	26,4	386	200,0	13,0	171,7	158,0	P
	11	3AL	51,0	356	356	1,1	27,8	386	200,0	13,0	101,5	138,0	F
	12	3AL	51,0	356	356	2,2	25,0	386	200,0	13,0	183,2	185,0	F
	13	3AL	51,0	406	406	1,1	24,9	386	200,0	13,0	107,4	145,0	F
	14	3AL	51,0	406	406	2,2	24,6	386	200,0	13,0	194,7	185,0	P
Damasceno [34]	L1A	2	89,3	85	85	1,22	41,3	600	240,0	13,0	265,5	188,5	P
	L2A	2	89,3	85	255	1,22	40,0	600	240,0	13,0	264,6	254,0	FP
	L3A	2	99,7	85	425	1,09	39,7	600	240,0	13,0	303,1	297,0	F
	L4A	2	98,6	85	595	1,1	40,4	600	240,0	13,0	295,5	325,0	F
	L1B	2	98,1	85	85	0,56	41,4	600	240,0	13,0	296,6	172,0	P
	L2B	2	90,5	85	255	0,61	42,0	600	240,0	13,0	273,9	194,5	P
	L3B	2	92,7	85	425	0,59	41,6	600	240,0	13,0	286,4	232,0	FP
	L4B	2	98,1	85	595	0,56	40,5	600	240,0	13,0	292,5	254,5	FP
Vilhena et.al. [31]	L1a	2	65,0	85	85	1,2	51,2	518	259,0	12,0	73,0	123,0	P
	L1b	2	65,0	85	85	1,4	51,2	518	259,0	12,0	89,0	122,0	P
	L3a	2	65,0	85	255	1,2	53,6	518	259,0	12,0	73,0	134,5	FP
	L3b	2	67,0	85	255	1,4	53,6	518	259,0	12,0	92,0	134,0	FP
	L5a	2	65,0	85	425	1,2	55,2	518	259,0	12,0	73,0	122,0	F
	L5b	2	65,0	85	425	1,4	55,2	518	259,0	12,0	89,0	124,5	F
Moraes Neto [35]	L1	3	87,0	85	85	1,4	39,4	602	255,3	13,0	453,0	224,0	P
	L2	3	87,5	85	255	1,2	39,8	602	255,3	13,0	422,0	241,0	P
	L3	3	86,5	85	425	1,3	40,9	602	255,3	13,0	473,0	294,0	P

P is the failure mode by punching; FP is the failure mode by ductile punching; and F is the failure mode by bending.

the slabs classified as failing by flexure were not considered in the statistical analysis presented below.

4. Results

Tables 3a and 3b show comparisons between experimental and theoretical results obtained for each code analyzed and also for the adaptations proposed. Figures 7 and 8 graphically show the accuracy of each code. In these figures, the results of the trend lines are indicated in blue and the red line marks the safety limit. Results below this line indicate unsafe theoretical predictions if compared to the experimental evidence.

In general, ACI 318 [5] presented conservative and disperse predictions if compared to the other codes. The average was of 1.45 and the coefficient of variation was of 17%. However, all the theoretical resistances estimated according to ACI were in favor of safety, with V_{exp}/V_{norma} greater than 1.0. The Eurocode 2 [4] showed satisfactory results, with a mean of 1.15 and 14% coefficient of variation. In the

Table 2 – (cont. 3)

Author	Slab	Type	d (mm)	c_{min} (mm)	c_{max} (mm)	ρ (%)	f'_c (MPa)	f_{ys} (MPa)	E_{sf} (GPa)	dg (mm)	P_{flex} (kN)	V_{exp} (kN)	Failure mode
Mouro [33]	L1	3	94,0	250	250	1,39	29,0	597	215,0	13,0	520,8	375,0	P
	L2	3	93,0	230	270	1,4	29,0	597	215,0	13,0	513,2	390,0	P
	L3	3	94,0	215	285	1,39	29,0	597	215,0	13,0	520,8	375,0	P
	L4	3	90,0	200	300	1,45	29,0	597	215,0	13,0	493,8	395,0	P
	L5	3	91,0	165	335	1,43	22,0	597	215,0	13,0	475,3	385,0	P
	L6	3	91,0	125	375	1,43	22,0	597	215,0	13,0	479,5	350,0	P
	L7	3	91,0	110	390	1,43	22,0	597	215,0	13,0	476,2	300,0	P
	L8	3	94,0	100	400	1,39	22,0	597	215,0	13,0	500,0	275,0	P
Regan [23]	I/2	3	77,0	200	200	1,2	23,4	500	200,0	10,0	374,5	176,0	P
	I/4	3	77,0	200	200	0,92	32,3	500	200,0	10,0	373,1	194,0	P
	I/6	3	79,0	200	200	0,8	21,9	480	200,0	10,0	250,0	165,0	P
	I/7	3	79,0	200	200	0,8	30,4	480	200,0	10,0	251,4	186,0	F
	II/1	3	200,0	250	250	0,98	34,9	530	200,0	20,0	2171,1	825,0	P
	II/2	3	128,0	160	160	0,98	33,3	485	200,0	20,0	812,5	390,0	P
	II/3	3	128,0	160	160	0,98	34,3	485	200,0	10,0	811,1	365,0	P
	II/4	3	64,0	80	80	0,98	33,3	480	200,0	20,0	198,3	117,0	P
	II/5	3	64,0	80	80	0,98	34,3	480	200,0	10,0	198,1	105,0	P
	II/6	3	64,0	80	80	0,98	36,2	480	200,0	5,0	198,1	105,0	P
Tomaszewicz [25]	ND65-1-1	3	275,0	200	200	1,5	64,3	500	200,0	16,0	5694,4	2050	P
	ND65-2-1	3	200,0	150	150	1,7	70,2	500	200,0	16,0	3333,3	1200	P
	ND95-1-1	3	275,0	200	200	1,5	83,7	500	200,0	16,0	5625,0	2250	P
	ND95-1-3	3	275,0	200	200	2,5	89,9	500	200,0	16,0	9600,0	2400	P
	ND95-2-1	3	200,0	150	150	1,7	88,2	500	200,0	16,0	3333,3	1100	P
	ND95-2-1D	3	200,0	150	150	1,7	86,7	500	200,0	16,0	3333,3	1300	P
	ND95-2-3	3	200,0	150	150	2,6	89,5	500	200,0	16,0	5178,6	1450	P
	ND95-2-3D	3	200,0	150	150	2,6	80,3	500	200,0	16,0	5208,3	1250	P
	ND95-2-3D+	3	200,0	150	150	2,6	98,0	500	200,0	16,0	5178,6	1450	P
	ND95-3-1	3	88,0	100	100	1,8	85,1	500	200,0	16,0	702,1	330	P
	ND115-1-1	3	275,0	200	200	1,5	112,0	500	200,0	16,0	5697,7	2450	P
	ND115-2-1	3	200,0	150	150	1,7	119,0	500	200,0	16,0	3333,3	1400	P
	ND115-2-3	3	200,0	150	150	2,6	108,1	500	200,0	16,0	5166,7	1550	P

P is the failure mode by punching; FP is the failure mode by ductile punching; and F is the failure mode by bending.

case of Eurocode, about 87% of results were found in favor of safety, taking up to 13% of unsafe results. Among all the results against the security, 84% refer to the slabs with rectangular columns. This indicates that Eurocode 2 equations require adjustments for these cases. In the case of NBR 6118 [3], the coefficient of variation was of 13%, similar to the one from Eurocode, but with a strong trend of unsafe results. The Brazilian code had a mean of 0.95 and overestimated the strength of 71% of the slabs of the database, indicating that it would be important to review the current recommendations. About 42% of its unsafe results refer to cases of slabs supported on square columns, which is one of the most basic situations in the design of buildings with flat slabs. This contrasts with the fact that the code was recently reviewed, but the punching shear recommendations were maintained, which are still based on CEB-FIP MC90 [16], neither reflecting the state of the art nor the advances obtained in the last two decades.

fib Model Code 2010 [6, 7] on its level I approach presented conservative results, with average of 1.45 and with ther higher coefficient of variation (26%), but with most of V_{exp}/V_{code} values above the reference value. Their results, compared with ACI 318 [5], were worse, with 11% of the slabs presenting theoretical predictions against safety. It should be noted, however, that the level I approach has a lower accuracy in the degree of precision, and its application is recommended for a pre-design of structures. The best results were observed for the level II of aproximation, with an average of 1.04 and coefficient of variation coefficient of 14%. However, there was obtained about 38% of unsafe results.

Applying the bending factor (λ) it was possible to enhance both the recommendations of NBR 6118 and MC2010. For MC2010, the average increased to 1.10, but the coefficient of variation decreased to 13%. Furthermore, the number of unsafe results was significantly reduced, changing from 38% to 21%. In the case of NBR 6118, the adoption of the bending factors coupled with the reduction of the coefficient of 0.18 to 0.16 improved significantly the code's predictions. The average was increased to 1.14, but the coefficient of variation decreased to 11%. However, the main quality gain is related to the number of unsafe results, which were to more acceptable levels. They reduced from 71% to only 10%.

Table 4 presents the evaluation of codes as a function of the adapted criteria from Collins [9]. According to this criteria, NBR 6118 [3] presented the higher penalty level (106 points), with 19% of the values in the second classification range (between 0.50 and 0.85), unfavorable in terms of safety. The Level of Approximation I from fib Model Code 2010 [6, 7] and ACI 318 [5] were also penalized, but in these cas-

Table 3a – Comparison between the experimental and theoretical results

Author	d (mm)	ρ (%)	f_c (MPa)	V_{exp}/V_{code} ACI MED	ACI COV	EC2 MED	EC2 COV	NB1 MED	NB1 COV
Forssel e Holmberg (19)	104 - 112	0,63 - 0,68	17	1,58	0,26	1,18	0,09	0,99	0,09
Elstner e Hognestad (20)	114	2,48	13 - 34	1,48	0,11	1,13	0,09	0,90	0,09
Mowrer e Vanderbilt (21)	51	1,1 - 2,2	15 - 28	1,64	0,11	1,32	0,07	0,85	0,07
Hawkins et. al (10)	117 - 120	0,76 - 1,12	23 - 32	1,15	0,08	0,99	0,06	0,85	0,07
Regan (22)	101 - 190	1,28	42 - 43	1,52	0,14	1,14	0,17	1,01	0,05
Regan (23)	64 - 200	0,8 - 1,2	21 - 36	1,34	0,12	1,21	0,10	0,95	0,06
Regan e Rezai-Jorabi (24)	79	1,51 - 1,54	29 - 31	1,50	0,05	1,12	0,01	0,86	0,01
Tomaszewicz (25)	88 - 275	1,5 - 2,6	64 - 119	1,69	0,10	1,13	0,08	1,04	0,07
Teng et al. (26)	102 - 160	1,07 - 1,81	33 - 40	1,38	0,14	1,09	0,14	0,95	0,09
Borges (27)	139 -164	1,23 -1,46	36 - 44	1,29	0,07	1,04	0,05	0,97	0,03
Lima Neto (28)	95 - 102	1,34 -1,44	41 - 47	1,47	0,08	1,26	0,08	1,03	0,09
Oliveira (8)	106 - 109	1,07 - 1,1	54 - 67	1,20	0,06	1,07	0,06	0,89	0,06
Al-Yousif e Regan (29)	80	0,98	21 - 23	1,36	0,12	1,13	0,17	0,86	0,17
Ferreira (30)	87 - 89	0,94 - 1,48	42 - 51	1,39	0,05	1,10	0,05	0,86	0,05
Vilhena et. al. (31)	65 - 67	1,2 - 1,4	51 - 55	1,32	0,01	1,12	0,04	0,80	0,04
Carvalho (32)	85 - 86	1,32	50 - 52	1,29	0,03	1,00	0,06	0,78	0,06
Mouro (33)	90 - 94	1,39 -1,45	22 - 29	1,84	0,11	1,50	0,11	1,20	0,11
Damasceno (34)	89 - 99	0,56 -1,22	39 - 42	1,23	0,13	1,10	0,02	0,88	0,02
Moraes Neto (35)	86 - 87	1,2 - 1,4	39 - 40	1,66	0,08	1,23	0,06	0,97	0,06
			Average	1,45	1,15	0,95	101	117	71
			COV (%)	17,25	14,06	12,92	8	16	5

es, because they presented many conservative predictions, having scores of 100 and 98 points, respectively. The MC2010 Level II and Eurocode 2 [4] (without adjustments) had the best performance according to this criteria, having 65 penalty and 43 points, respectively.
If the changes suggested were implemented, NBR 6118 [3] could present significant improvements. Its penalty points would decrease to 42 points, with most of the results in the range where the penalty is equal to zero (between 0.85 and 1.15). If the bending factor (λ) were applied in the fib Model Code 2010 [6, 7] Level II, the best performance would be achieved in terms of design, with only 40 penalty points, the lowest among all tested hypotheses.
Figures 9 and 10 show the trend lines for each theoretical method according to the rectangularity index of the columns. It is clear to all design methods that there is a declining trend in V_{exp}/V_{code} as a function of the rectangularity index of the column. In the case of NBR 6118 [3], Eurocode 2 [4] and fib Model Code 2010 [6, 7], there is a tendency to overestimate the resistance of slabs supported on rectangular columns, especially in the case of the Brazilian code.
The changes proposed to the NBR 6118 [3] and the fib Model Code 2010 [6, 7] level II corrected this trend of unsafe results, especially in the case of the Brazilian code.

5. Conclusions

This paper used results of experimental tests of 131 slabs to evaluate different recommendations for the punching shear design of slab-column connections. The variables evaluated were the boundary conditions and the rectangularity index of the columns. It was observed that, generally, the recommendations presented by ACI 318 [5] and fib Model Code 2010 [6, 7] Level I are in favor of safety, but are conservative, indicating the possibility of adjustments to avoid an exagereted level of security. Eurocode 2 [4] and the MC2010 Level II showed satisfactory results, with MC2010 beeing slightely more accurate, but with 43% of unsafe results, against 20% for Eurocode.
The worst results were observed for the NBR 6118 [3], which provided 77% of unsafe results. The proposed adaptations for both the MC2010 Level II and for the NBR 6118, proved to be effective, improving significantly the quality of results. In the specific case of NBR 6118, the adjustments reduced the number of results against the safety. They also were able to correct the unsafe trend observed, that showed that unsafe predictions were increasing proportionaly to increments in the rectangularity index of the column.

Table 3b – Comparison between the experimental and theoretical results

Author	f_c (MPa)	V_{exp}/V_{code} NB1*+ λ MED	NB1*+ λ COV	MC10 I MED	MC10 I COV	MC10 II MED	MC10 II COV	MC10 II+ λ MED	MC10 II+ λ COV
Forssel e Holmberg (19)	17	1,19	0,09	1,02	0,14	0,97	0,06	1,03	0,05
Elstner e Hognestad (20)	13 - 34	1,11	0,07	1,29	0,12	1,08	0,07	1,17	0,09
Mowrer e Vanderbilt (21)	15 - 28	1,02	0,06	1,11	0,20	1,03	0,13	1,09	0,12
Hawkins et. al (10)	23 - 32	1,11	0,06	1,04	0,07	0,95	0,07	1,09	0,11
Regan (22)	42 - 43	1,13	0,08	1,35	0,13	1,20	0,10	1,18	0,13
Regan (23)	21 - 36	1,13	0,06	1,54	0,14	1,02	0,11	1,06	0,11
Regan e Rezai-Jorabi (24)	29 - 31	1,00	0,01	1,38	0,05	1,08	0,04	1,11	0,05
Tomaszewicz (25)	64 - 119	1,22	0,07	1,94	0,09	1,05	0,05	1,07	0,05
Teng et al. (26)	33 - 40	1,15	0,08	1,30	0,22	0,99	0,17	1,06	0,16
Borges (27)	36 - 44	1,16	0,04	1,84	0,11	1,06	0,06	1,11	0,05
Lima Neto (28)	41 - 47	1,22	0,09	1,58	0,09	1,17	0,09	1,21	0,09
Oliveira (8)	54 - 67	1,09	0,05	1,39	0,19	0,92	0,10	0,99	0,09
Al-Yousif e Regan (29)	21 - 23	1,14	0,08	0,95	0,15	0,91	0,15	1,06	0,06
Ferreira (30)	42 - 51	0,98	0,05	1,49	0,05	1,13	0,06	1,13	0,06
Vilhena et. al. (31)	51 - 55	0,96	0,04	1,19	0,01	1,31	0,06	1,37	0,06
Carvalho (32)	50 - 52	0,96	0,02	0,75	0,24	0,81	0,14	0,87	0,06
Mouro (33)	22 - 29	1,44	0,11	1,86	0,11	1,29	0,11	1,36	0,11
Damasceno (34)	39 - 42	1,01	0,06	1,28	0,22	0,99	0,15	1,01	0,13
Moraes Neto (35)	39 - 40	1,15	0,04	1,48	0,27	1,05	0,16	1,10	0,14
	Average	1,14	1,14	1,45	1,45	1,04	1,04	1,10	1,10
	COV (%)	11,80	11,80	26,30	26,30	14,10	14,10	13,20	13,20

It should also be noted that the proposals made are simple to implement in terms of calculation. It is hoped that these and other work can motivate a deep discussion on the current recommendations for the design of flat slabs in Brazil, in order to allow the Brazilian code to reflect the state of the art and the progress obtained through many researches conducted in Brazil and abroad.

6. Acknowledgements

The authors would like to acknowledge FAPESPA, CNPq and CAPES for the financial support in all steps of this research.

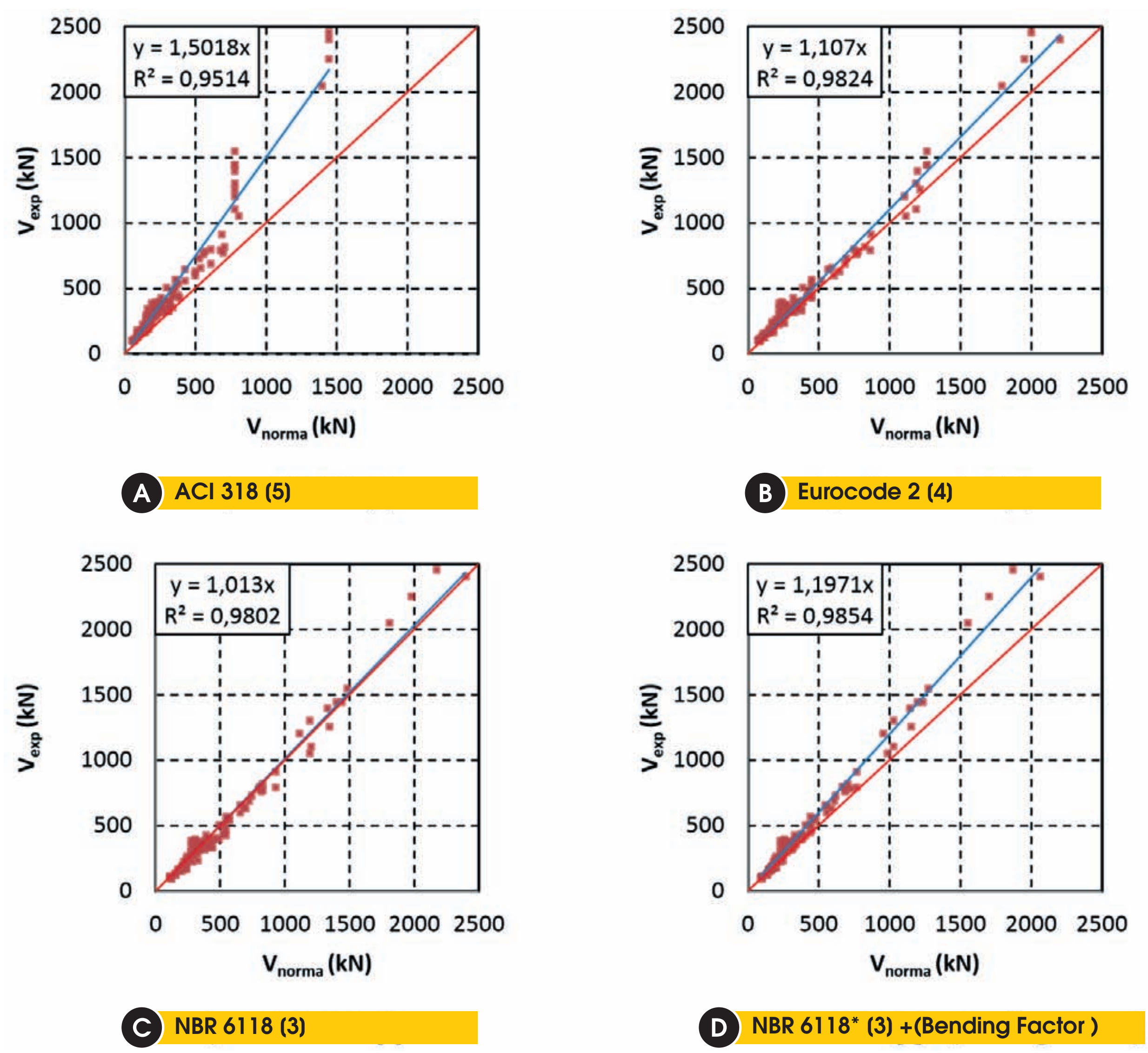
7. References

[1] MIDDLETON, R., Christchurch Earthquake Reconnaissance Part 1: The First 3 days [2011]. 1 fotografia, color. Disponível em:<http://reidmiddleton.wordpress.com/2011/03/14/5/>. Acesso em: 13 jan. 2013.

[2] FERREIRA, M. P., OLIVEIRA, D. R. C., Numerical Analysis of RC Flat Slabs under Flexure and Punching. IBRACON Structural Journal, Vol. 2, No. 3, 2006, pp. 371–392.

[3] ASSOCIAÇÃO BRASILEIRA DE NORMAS TÉCNICAS. NBR 6118 – Projeto de Estruturas de Concreto. Rio de Janeiro, 2007.

Figure 7 – Tendency of the results according to different theories



[4] Eurocode 2, Design of Concrete Structures—Part 1-1: General Rules and Rules for Buildings, CEN, EN 1992-1-1, Brussels, Belgium, 2004, 225 pp.

[5] ACI Committee 318, Building Code Requirements for Structural Concrete (ACI 318M-11) and Commentary, American Concrete Institute, Farmington Hills, Michigan, 2011.

[6] Fédération Internationale du Béton (fib), Model Code 2010, final draft, vol. 1. fib, Bulletin 65, Lausanne, Switzerland, 2012, Vol. 2, 350 pp.

[7] Fédération Internationale du Béton (fib), Model Code 2010, final draft, vol. 2. fib, Bulletin 66, Lausanne, Switzerland, 2012, Vol. 2, 370 pp.

[8] OLIVEIRA, D. R. C., Análise experimental de lajes cogumelo de concreto armado com pilares retangulares. Tese de Doutorado, Publicação E.TD – 001A/2003, Departamento de Engenharia Civil, Universidade de Brasília, Brasília, DF, 2003. 214p.

[9] COLLINS, M.P. (2001). Evaluation of shear design procedures for concrete structures. A Report prepared for the CSA technical committee on reinforced concrete design.

[10] HAWKINS, N.M., FALLSEN, H.B., HINOJOSA, R.C.,

Figure 8 – Tendency of the results according to different theories

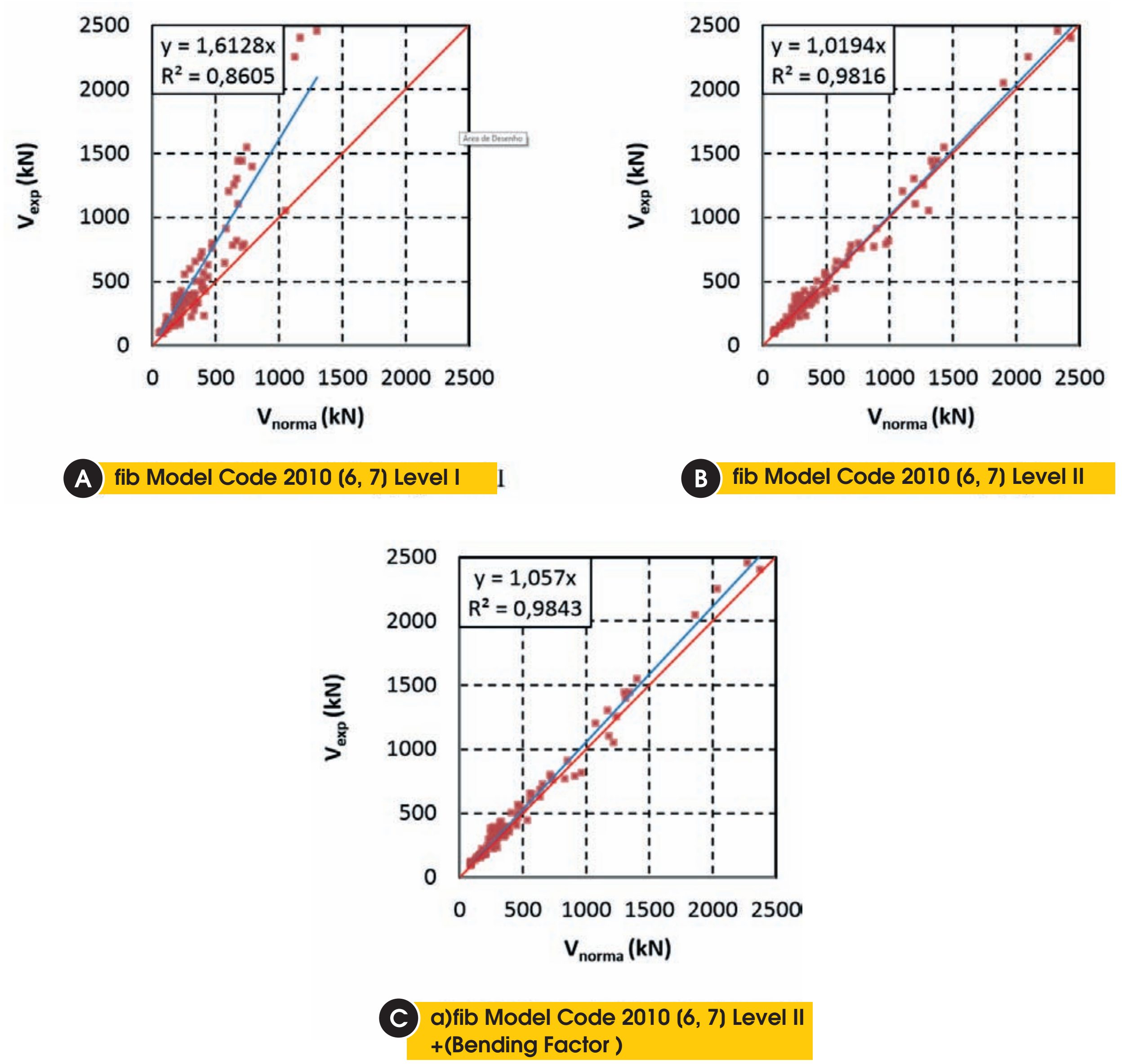


Table 4 – Classification according to the criteria of Collins (9)

		< 0,50	(0,50:0,85)	(0,85:1,15)	(1,15:2,00)	≥ 2,00	TOTAL
ACI	N° of slabs	0	0	9	94	2	105
	Total penalty	0	0	0	94	4	98
EC2	N° of slabs	0	0	62	43	0	105
	Total penalty	0	0	0	43	0	43
NB1	N° of slabs	0	20	79	6	0	105
	Total penalty	0	100	0	6	0	106
NB1*+ λ	N° of slabs	0	0	63	42	0	105
	Total penalty	0	0	0	42	0	42
MC10 I	N° of slabs	0	3	26	67	9	105
	Total penalty	0	15	0	67	18	100
MC10 II	N° of slabs	0	9	76	20	0	105
	Total penalty	0	45	0	20	0	65
MC10 II+ λ	N° of slabs	0	2	73	30	0	105
	Total penalty	0	10	0	30	0	40

Influence of Column Rectangularity on the Behavior of Flat Plate Structures, ACI Special Publication, Cracking, Deflection, and Ultimate Load of Concrete Slab Systems, Vol. SP-30, No. 6, 1971, pp. 127–146.

[11] OLIVEIRA, D. R. C.; REGAN, P. E.; MELO, G. S. S., Punching Resistance of RC Slabs with Rectangular Columns. Magazine of Concrete Research, Vol. 56, No. 3, London, 2004, pp. 123–138.

[12] MUTTONI, A., and SCHWARTZ, J., Behavior of Beams and Punching in Slabs without Shear Reinforcement, IABSE Colloquium, Vol. 62, Zurich, Switzerland, 1991, pp. 703–708.

[13] MUTTONI, A., Punching Shear Strength of Reinforced Concrete Slabs without Transverse Reinforcement, ACI Structural Journal, Vol. 105, No. 4, 2008, pp. 440–450.

[14] FERNÁNDEZ RUIZ, M., MUTTONI, A., Applications of the critical shear crack theory to punching of R/C slabs with transverse reinforcement, ACI Structural Journal, Vol. 106, No. 4, 2009, pp. 485–494.

[15] SAGASETA, J., MUTTONI, A., FERNÁNDEZ RUIZ, M., TASSINARI, L., Non axis-symmetrical punching shear around internal columns of RC slabs without transverse reinforcement, Magazine of Concrete Research, Vol. 63, No. 6, 2011, pp. 441–457.

[16] Comité Euro-International du Béton. CEB-FIP Model Code 1990. London, Thomas Telford, 1993.

[17] SACRAMENTO, P. V. P., FERREIRA, M. P., OLIVEIRA, D. R. C., MELO, G. S. S. A., Punching strength of reinforced concrete flat slabs without shear reinforcement, IBRACON Structures and Materials Journal, Vol. 5, No. 5, 2012, pp. 659–691.

[18] OLIVEIRA, M. H., PEREIRA FILHO, M. J. M., OLIVEIRA, D. R. C., FERREIRA, M. P., MELO, G. S. S. A., Punching resistance of internal slab-column connections with double-headed shear studs, IBRACON Structures and Materials Journal, Vol. 6, No. 5, 2013, pp. 681–714.

[19] FORSSEL, C., HOLMBERG, A., Stämpellast pa plattor av betong. Betong, 1946, Vol. 31, No. 2, pp. 95–123.

[20] ELSTNER, R. C., e HOGNESTAD, E., Shearing Strength of Reinforced Concrete Slabs. Journal of the American Concrete Institute, Proceedings, Vol. 53, No. 1, Jul. 1956, pp. 29–58.

[21] MOWRER, R. D., VANDERBILT, M. D., Shear Strength of Lightweight Aggregate Reinforced Concrete. ACI Journal, 1967, Vol. 64, No. 11, November, pp. 722–729.

[22] REGAN, P. E., Punching Shear in Prestressed Concrete Slab Bridges. Polytechnic of Central London, January 1983.

[23] REGAN, P. E., Symmetric Punching of Reinforced Concrete Slabs. Magazine of Concrete Research, Vol. 38, No. 136, Sep. 1986, pp 115–128.

[24] REGAN, P. E., REZAI-JORABI, H., Shear Resistance of One-way Slabs Under Concentrated Loads. ACI Structural Journal, 1988, Vol. 85, No. 2, Mar-Apr, pp. 150–157.

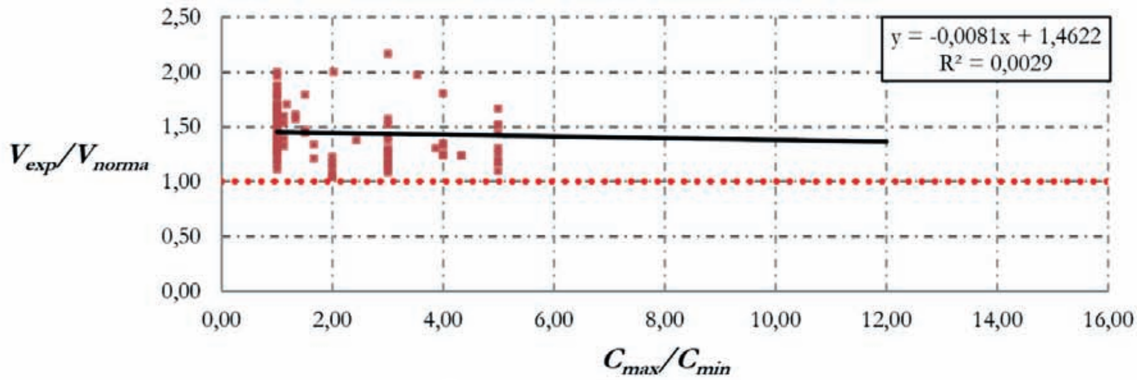
[25] TOMASZEWICZ, A., High-Strength Concrete. SP2 – Plates and Shells. Report 2.3 Punching Shear Capacity of Reinforced Concrete Slabs. Report No. STF70 A93082, SINTEF Structures and Concrete, Trondheim, 1993, 36pp.

[26] LEONG, K. K., TENG, S., Punching Shear Strength of Slabs with Openings and Supported on Rectangular Columns, Nanyang Technological University, Singapore, August 2000.

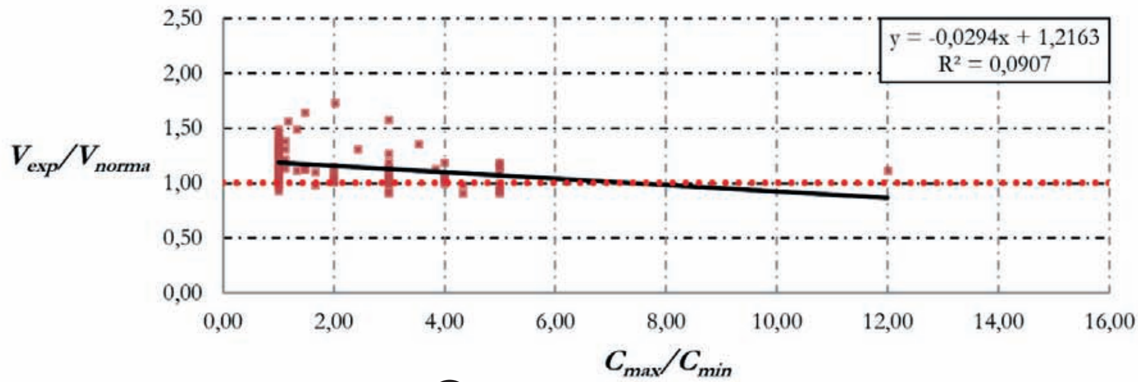
[27] BORGES, L. L. J., Comportamento ao Puncionamento de Lajes Cogumelo com Pilares Retangulares e Furos de Grandes Dimensões, Tese de Doutorado, Departamento de Engenharia Civil e Ambiental, Universidade de Brasília, DF, 2004.

[28] LIMA NETO, A. F., 2003. Análise experimental na fase pós-puncionamento de lajes cogumelo de concreto armado.

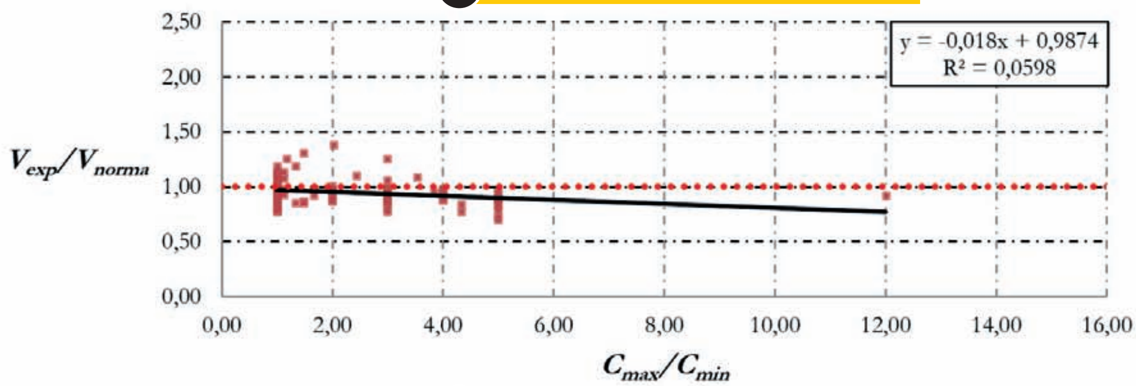
Figure 9 – Tendency of the results according to different theories



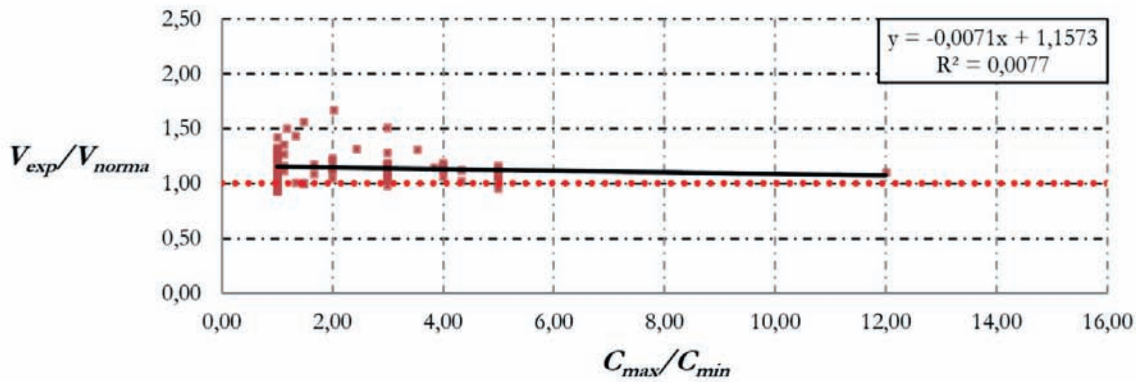
A ACI 318 (5)



B Eurocode 2 (4)

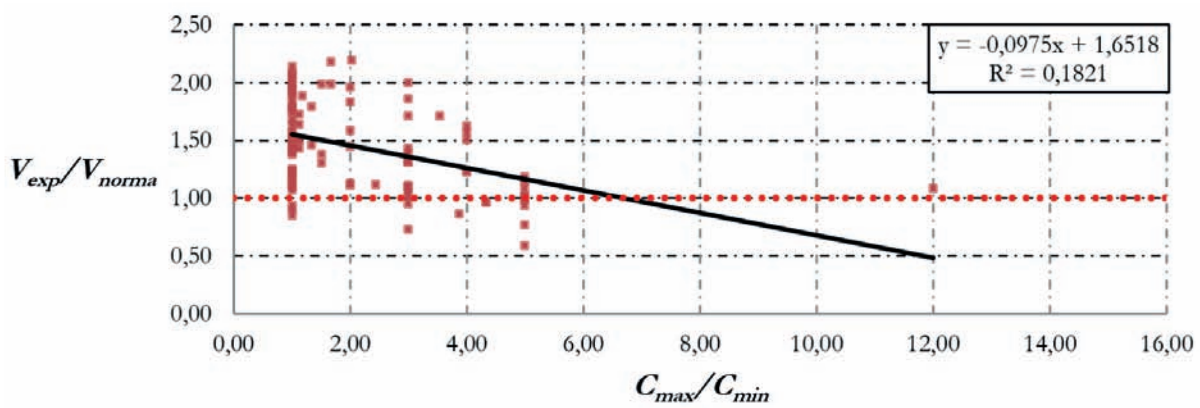


C NBR 6118 (3)

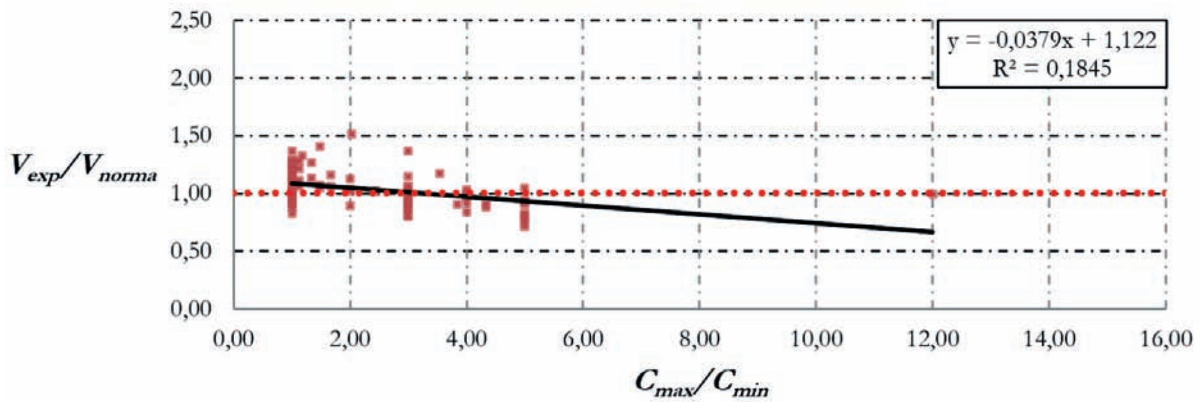


D NBR 6118* (3) +Bending Factor

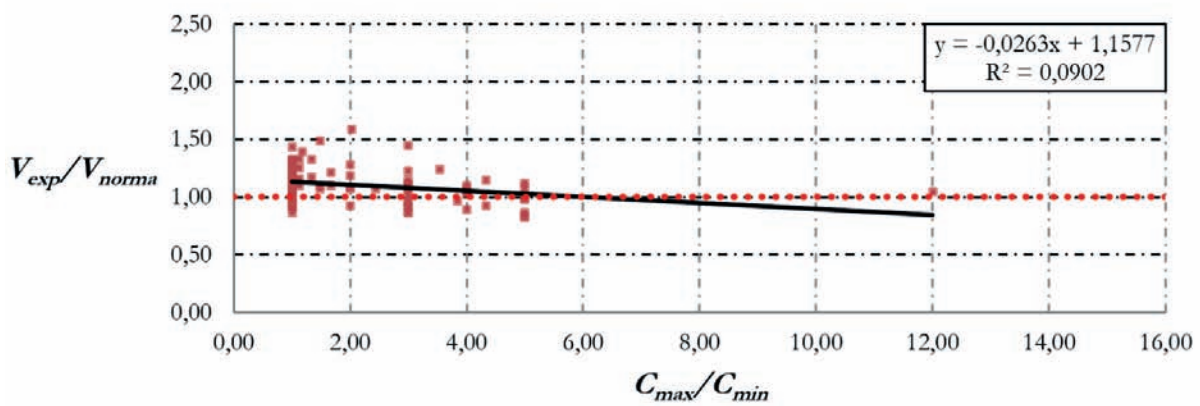
Figure 10 – Tendency of the results according to different theories



A fib Model Code 2010 (6, 7) - Level I



B fib Model Code 2010 (6, 7) - Level II



C fib Model Code 2010 (6, 7) - Level II + Bending Factor

Dissertação de Mestrado. Publicação E. DM. 008A/2003. Departamento de Engenharia Civil e Ambiental, Universidade de Brasília. Brasília, DF.

[29] AL-YOUSIF, A. T., REGAN, P. E., *Punching Resistances of RC Slabs Supported by Large and/or Elongated Columns*. The Structural Engineer, London, Mar. 2003, p. 30-34.

[30] FERREIRA, M. P. Análise experimental de lajes lisas unidirecionais de concreto armado ao puncionamento simétrico ou assimétrico. Dissertação de Mestrado, Publicação CDD-22. Ed. 624.18342, Departamento de Engenharia Civil, Universidade Federal do Pará, Belém, PA, 2006.

[31] VILHENA, A. E. R., NAHUN, K. A. R., PERES, M. D., OLIVEIRA, D. R. C., Análise experimental de lajes unidirecionais com pilares retangulares ao puncionamento simétrico. In: 48º Congresso Brasileiro do Concreto, 2006. Rio de Janeiro.

[32] CARVALHO, A. S. de Análise Experimental de Laje Lisa de Concreto Armado de Alta Resistência com Metacaulim Apoiadas em Pilares Retangulares e Armada à Punção. Departamento de Engenharia Civil, Universidade Federal do Pará, Dissertação de Mestrado, Belém, 2006.

[33] MOURO, V.C, Punção em lajes cogumelo: estudo de retangularidade dos pilares - 2006 dissertação de mestrado (Universidade Federal de Goiás Escola de Engenharia Civil).

[34] DAMASCENO, L.S. R, 2007. Análise experimental de lajes lisas unidirecionais de concreto armado com pilares retangulares ao puncionamento. Dissertação de Mestrado. publicação CDD - 22. Ed. 624.1834. Universidade Federal do Pará, Instituto de Tecnologia, Departamento de Engenharia Civil, Programa de Pós-Graduação em Engenharia Civil, Belém, 2007.

[35] MORAES NETO, B. N., Análise Experimental de Lajes Lisas Bidirecionais de Concreto Armado com Estribos Inclinados e Pilares Retangulares Sujeitas a Puncionamento Simétrico Belém, Pará, 2007.

[36] MORAES NETO, B.N. (2013). Comportamento à Punção de Lajes Lisas em Concreto Reforçado com Fibras de Aço sob Carregamento Simétrico. Tese de Doutorado em Estruturas e Construção Civil, Publicação E.TD-001A/13 Departamento de Engenharia Civil e Ambiental, Universidade de Brasília, Brasília, DF, 343 p.

[37] ASSOCIAÇÃO BRASILEIRA DE NORMAS TÉCNICAS. NBR 6118 – Projeto de Estruturas de Concreto. Rio de Janeiro, 2014.